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REVIEW OF METHODS FOR ANALYZING PUMP-TEST DATA



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Abstract

Quantitative data on hydraulic characteristics of aquifers including transmissivity and storativity are essential to the understanding and solution of aquifer problems and the proper evaluation and utilization of ground water resources.

The methods of pump test analysis reviewed in this report has been classified as traditional methods, computer based methods and new graphical methods.

1.0 INTRODUCTION

The solution to the problem of ground water flow requires a proper knowledge of the hydraulic characteristics of aquifers. Therefore, it becomes essential to determine the aquifer parameters prior to planning and use of the ground water resources in an area.

Analyses of pump test data yield values of aquifer parameters such as transmissivity and storativity of the aquifer. There are two general types of analyses available for determination of aquifer parameters namely steady state or equilibrium method and transient or non equilibrium method. The principal difference between the two methods is that the unsteady methods permits analysis of ground water conditions which change with time and involve storage, whereas the steady state method does not.

Now a days, there are several methods available for the evaluation of the pumping test data. In this report, some selected methods for determining aquifer parameters from analysis of pump test data have been discussed which are classified as, (1) Traditional methods, (2) Computer based methods and (3) New graphical methods, whose brief description are given below.

Under the head traditional methods, the methods reviewed are, Thiem method, Theis method, Chow method, Cooper and Jacob method and Theis recovery method. Under the head computer based methods, the following methods are reviewed, (1) New automated technique for the derivation of aquifer parameters in a non leaky confined aquifer, (2) Application of microcomputer in the analysis of pump test data in confined aquifer, (3) Programmable hand calculator programs for pumping test analysis by least squares method, (4) Methods of non linear regression analysis based on the principles of least squares, (5) Numerical methods of pumping test analysis using microcomputers, (6) Algorithm for Theis solution of pumping test data. Under the head new graphical methods, the methods reviewed are, (1) An alternate procedure for analyzing aquifer tests using the Theis non equilibrium solution, (2) A simplified graphical solution of the Theis equation, (3) New approach for analysis of aquifer test data, (4) A new graphical method for identification of aquifer parameters.

2.0 TRADITIONAL GRAPHICAL METHODS

The following methods have been discussed.

1. Thiem method;
2. Theis method;
3. Chow method;
4. Cooper and Jacob method, and ;
5. Theis recovery method.

Assumptions common to all the above methods are given below.

- i) The aquifer has a seemingly infinite areal extent,
- ii) The aquifer is homogeneous, isotropic and of uniform thickness over the area of influenced by the pumping test,
- iii) Prior to pumping, the piezometric surface and /or phreatic surface are nearly horizontal over the area influenced by the pumping test,
- iv) The aquifer is pumped at a constant discharge rate,
- v) The pumped well penetrates the entire aquifer and thus receives water from the entire thickness of the aquifer by horizontal flow,

2.1 STEADY STATE FLOW IN CONFINED AQUIFER

2.1.1 Thiem's Method:

Thiem(1906) utilized two or more piezometers to determine the hydraulic conductivity of an aquifer using the following equation

$$Q = \frac{2\pi T(h_2 - h_1)}{\ln(r_2/r_1)} \quad (2.1)$$

Where,

Q is the pumped discharge of well,

r_1 and r_2 are the respective distances of the piezometers from the pumped well,

h_1 and h_2 are the respective water levels in the piezometers,
T is the Transmissivity of the aquifer,

In terms of drawdown, eq. (2.1) is usually written as,

$$Q = \frac{2\pi T(s_1 - s_2)}{\ln(r_2/r_1)} \quad (2.2)$$

Where,

s_1 and s_2 are the respective steady state drawdowns in the piezometers,

There are two procedures for finding the value of Transmissivity in this method. The procedures are given below.

- (i) Substitute the values of the steady state drawdowns of the two piezometers in to eq.(2.1), together with the corresponding values of r and the known values of Q , and solve for T .

Repeat this procedure for all possible combinations of piezometers. Practically, the calculations give more or less different values of T . The average value is then calculated.

- (ii) A graph is prepared by Plotting on a single logarithmic paper, the observed steady state drawdown s of each piezometer against the distance r between the pumped well and the piezometer, the drawdown on the vertical axis on a linear scale and the distance r on the horizontal axis (on logarithmic scale) (Fig.1). The straight line fitting through the plotted points is known as distance drawdown curve.

Determine the slope of this line ' Δs ' i.e.; the difference of maximum drawdown per log cycle of r , giving $r_2/r_1 = 10$

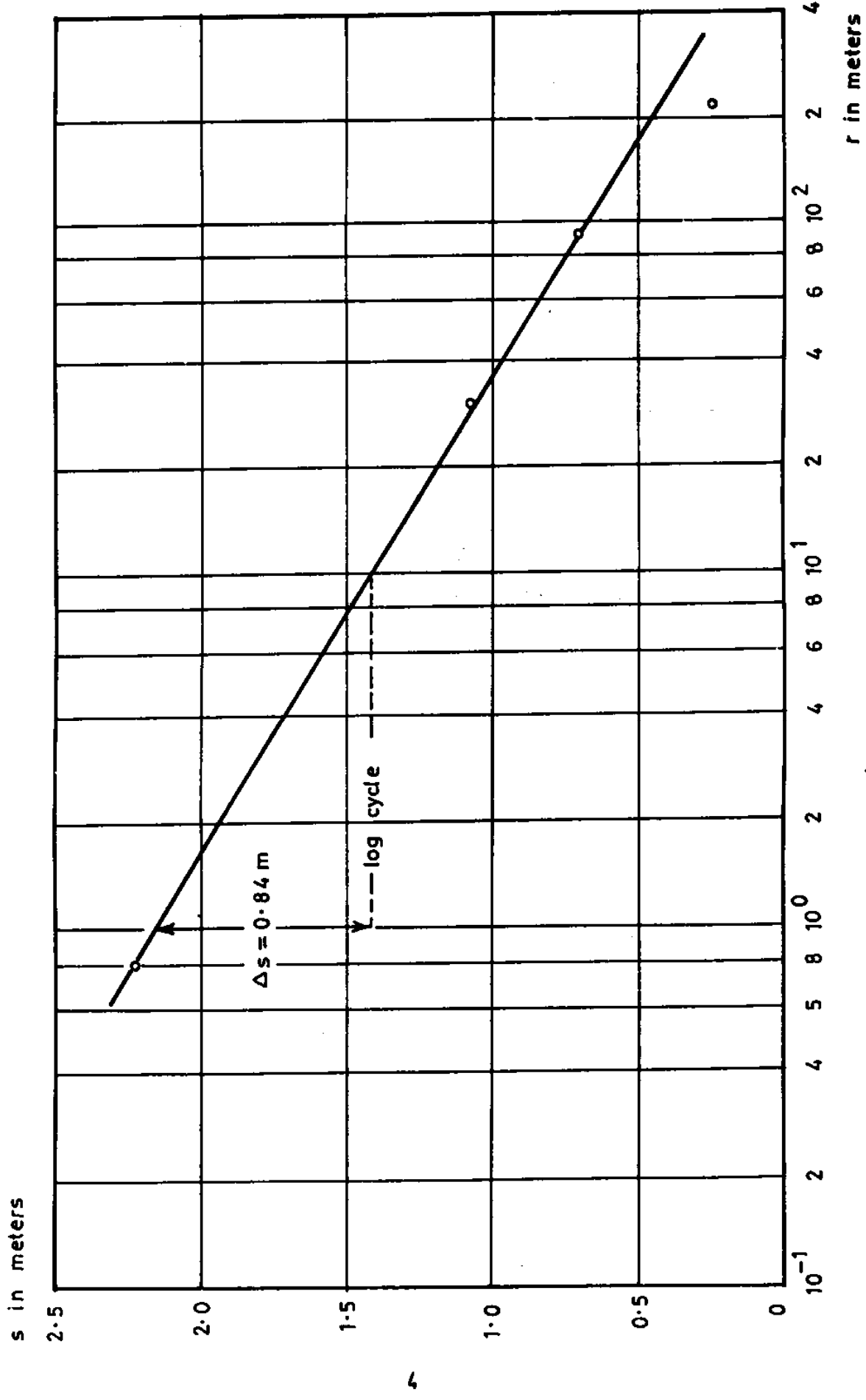


Fig.1: Analysis of data from pumping test 'Oude Korendijk' with the Thiem method, Procedure II

So, $\log_{10} \left(\frac{r_2}{r_1} \right) = 1$, thus equation (2.2) becomes

$$Q = \frac{2\pi T}{2.303} \Delta s \quad (2.3)$$

Substituting the numerical values of Q & Δs in to eq.(2.3), we find the corresponding values of 'T'. In this method Steady state is defined as the situation where variation of the drawdown with respect to time are negligible, or where the hydraulic gradient has become constant. But the true steady state, i.e.; zero drawdown variations with time is not possible in a confined aquifer. The drawback of the Thiem method is that it needs steady state drawdowns and considerable time is required for the flow to reach steady state.

Besides the assumptions listed under section 2.0 the following additional assumptions are made below.

- (i) The aquifer is confined.
- (ii) The flow to the well is in steady state.

2.1.2 Unsteady state flow in confined aquifer

Theis(1935) developed a non steady state formula which introduces the time factor and the storage coefficient. The nonsteady state or Theis eq., which was derived from the analogy between the ground water flow and the conduction of heat, may be written as

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} dy \quad (2.4)$$

or,

$$s = \frac{Q}{4\pi T} W(u) \quad (2.5)$$

Where,

$$u = \frac{r^2 S}{4Tt} \quad (2.6)$$

and consequently,

$$S = \frac{4Ttu}{r} \quad (2.7)$$

In which,

s is the drawdown measured in a piezometer at distance ' r ' from the pumped well,

Q is the constant well discharge,

S is the dimensionless coefficient of storage,

T is the transmissivity of the aquifer,

t is the time since pumping started.

$W(u)$ in eq.(2.5) is exponential integral Which is generally, read as 'Well function of ' u ' or 'Theis well function' and can be expressed as

$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2.2} + \frac{u^3}{3.3} - \dots$$

If ' s ' can be measured for one or more values of ' r ' and for several values of t , and if the well discharge Q is known, S & T can be determined from eq.(2.5) & (2.7). The presence of two unknowns and the nature of the exponential integral makes it impossible to solve eq. (2.4). However, several approximate graphical solutions have been developed, to determine T and S , which are given below.

THEIS METHOD

In this method, a type curve of the Theis well function is prepared on double logarithmic paper by plotting values of $W(u)$ against the argument u , or the values of

$W(u)$ against $1/u$ is plotted (Annex.1). But generally type curve between $W(u)$ and $1/u$ is preferred for convenience. Now, plot the values of s against t/r^2 on another sheet of logarithmic paper of the same scale as that used for the Theis type curve.

At a constant discharge Q the drawdown s is related to t/r^2 in a manner which is similar to the relation of $W(u)$ to $1/u$, and the curve of the observed data will be similar to the Theis type curve. Place the observed data plot over the type curve and, keeping the coordinate axes of both the data plot and the type curve parallel to each other, locate the position of best match between the data plot and the type curve.

Select an arbitrary point 'A' on the overlapping portion of the two sheets of graph paper and determine the coordinates of $W(u)$, $1/u$, s & t/r^2 for this match point. It is not necessary to locate the match point along the type curve. But the match point should be selected such that where the coordinate of the type curve are $W(u)=1$, and $1/u=10$ (Fig.2).

Now substitute the values of $W(u)$, s & Q in to eq. 2.5

$$T = \frac{Q}{4\pi s} W(u) \quad \text{and solve for } T$$

then calculate 'S' by substituting the values of T , t/r^2 and u in to eq. 2.7

$$S = 4T(t/r^2)u$$

Assumptions for Theis method:

Besides the assumptions listed under section 2.0, the following additional assumptions are made below.

- (i) The aquifer is confined.
- (ii) The flow to the well is in unsteady state or the drawdown differences with time are not negligible.
- (iii) The water removed from storage is discharged instantaneously with decline of head.
- (iv) The diameter of the well is very small i.e.; the storage in the well is neglected.

ANNEXURE 1. TABLE OF VALUES OF $W(u)$ CORRESPONDING TO VALUES OF u AND $1/u$

u	N	$W(u)$	$n(1)$	$n(2)$	$n(3)$	$n(4)$	$n(5)$	$n(6)$	$n(7)$	$n(8)$	$n(9)$	$n(10)$
1.000	1.0	2.194(-1)	1.823	4.038	6.332	8.633	1.094(1)	1.324(3)	1.554(1)	1.784(1)	2.015(1)	2.245(1)
0.833	1.2	1.564(-1)	1.660	3.858	6.149	8.451	1.075(1)	1.306(1)	1.536(1)	1.766(1)	1.996(1)	2.227(1)
0.666	1.5	1.000(-1)	1.465	3.637	5.927	8.228	1.053(1)	1.283(1)	1.514(1)	1.744(1)	1.974(1)	2.204(1)
0.500	2.0	4.890(-2)	1.223	3.355	5.639	7.940	1.024(1)	1.255(1)	1.485(1)	1.715(1)	1.945(1)	2.176(1)
0.400	2.5	2.491(-2)	1.044	3.137	5.417	7.717	1.002(1)	1.232(1)	1.462(1)	1.693(1)	1.923(1)	2.153(1)
0.333	3.0	1.305(-2)	9.057(-1)	2.959	5.235	7.535	9.837	1.214(1)	1.444(1)	1.674(1)	1.905(1)	2.135(1)
0.286	3.5	6.970(-3)	7.942(-1)	2.810	5.081	7.381	9.683	1.199(1)	1.429(1)	1.659(1)	1.889(1)	2.120(1)
0.250	4.0	2.772(-3)	7.324(-1)	2.681	4.948	7.247	9.550	1.185(1)	1.415(1)	1.646(1)	1.876(1)	2.106(1)
0.222	4.5	2.073(-3)	6.253(-1)	2.568	4.831	7.130	9.432	1.173(1)	1.404(1)	1.634(1)	1.864(1)	2.094(1)
0.200	5.0	1.148(-3)	5.598(-1)	2.468	4.726	7.024	9.326	1.163(1)	1.393(1)	1.623(1)	1.854(1)	2.084(1)
0.166	6.0	3.601(-4)	4.544(-1)	2.295	4.545	6.842	9.144	1.145(1)	1.375(1)	1.605(1)	1.835(1)	2.066(1)
0.142	7.0	1.155(-4)	3.738(-1)	2.151	4.392	6.688	8.990	1.129(1)	1.360(1)	1.590(1)	1.820(1)	2.050(1)
0.125	8.0	3.767(-5)	3.106(-1)	2.027	4.259	6.555	8.856	1.116(1)	1.346(1)	1.576(1)	1.807(1)	2.037(1)
0.111	9.0	1.245(-5)	2.602(-1)	1.919	4.142	6.437	8.739	1.104(1)	1.334(1)	1.565(1)	1.795(1)	2.025(1)

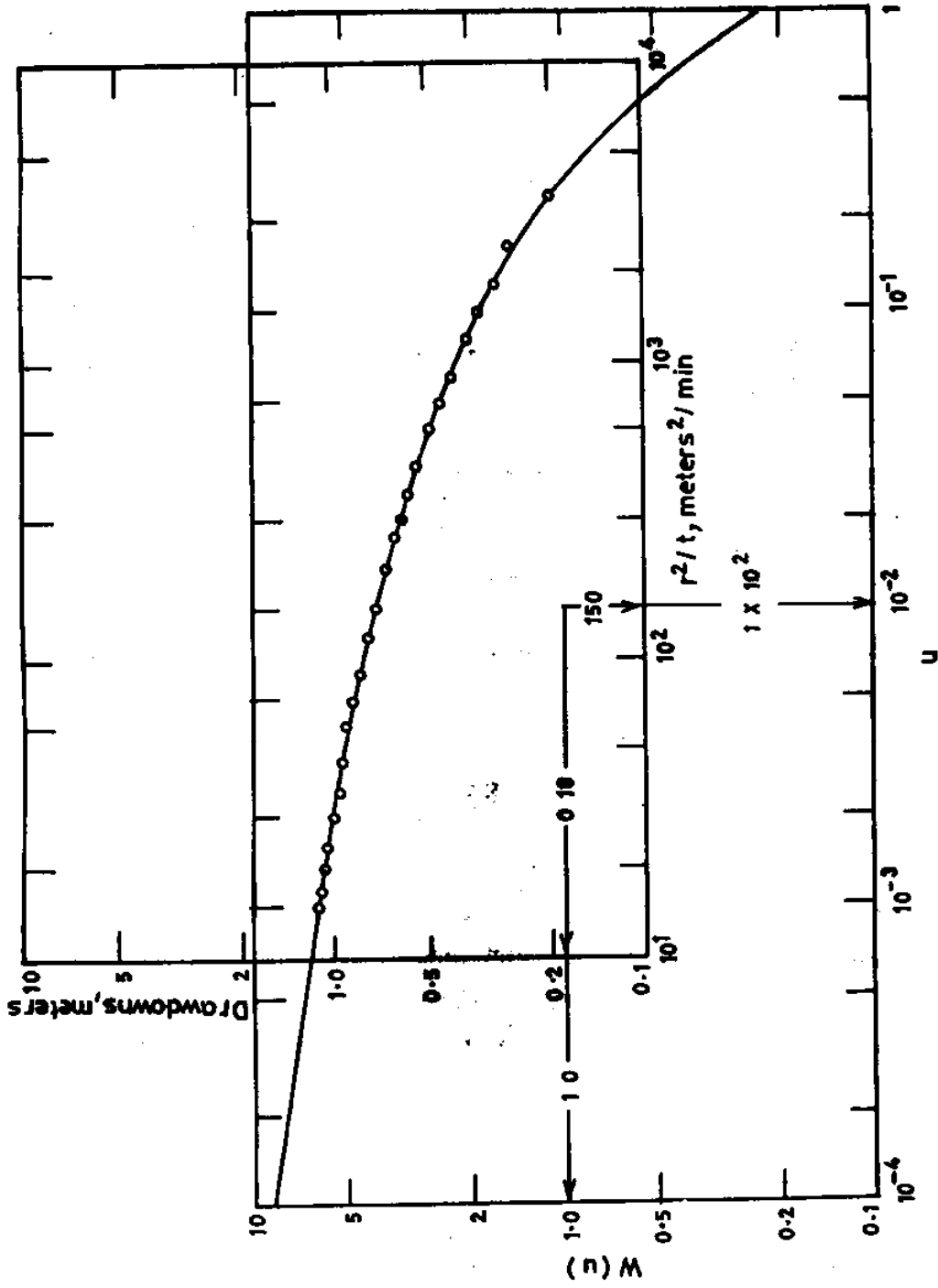


Fig. 2 Theis method of superposition for solution of the nonequilibrium equation.

CHOW'S METHOD:

Chow(1952) developed a method based on the Theis equation in which the curve fitting of the Theis method and restriction of small values of r and large value of t is avoided. The assumptions and conditions for this method are same as for the Theis method. He introduced the function,

$$F(u) = \frac{W(u) e^u}{2.303} \quad (2.8)$$

Find the value of $W(u)$ and u corresponding with the drawdown s measured at a certain moment t . The relation between $F(u)$, $W(u)$ & u is given in (Annex.ii) and $F(u)$ can easily be calculated from the procedure given below.

Plot for one of the piezometers the drawdown s versus the corresponding time t on single logarithmic paper (t on logarithmic scale). Select an arbitrary point 'A' on the curve through the plotted points and draw through 'A' a tangent to the curve. Read on the s -axis the drawdown value for point A, S_A and the slope of the tangent line i.e.; the drawdown difference per log cycle of time (ΔS_A), (Fig.3). Then calculate the value of $F(u)$ for the point A from

$$F(u) = \frac{S_A}{\Delta S_A} \quad (2.9)$$

Knowing the value of $F(u)$, find the corresponding value of $W(u)$ & u from the Annex (ii). Record the value of 'ta' from time axis of the observed data curve and substitute the appropriate numerical values into eq. (2.5) & (2.7) and solve for T and S . If $F(u) > 2.0$, then, $W(u) = 2.3 * F(u)$ and u is subsequently obtained from Annex (i)

COOPER AND JACOB METHOD:

Cooper and Jacob(1946) method is also based on the Theis formula, however, the conditions for its application are somewhat more restricted than for the Theis & Chow method. This method is applicable for small value of u (i.e.; $u < 0.01$). The exponential integral in the Theis eq. can be expanded in a convergent series, so that the drawdown 's' may be written as,

ANNEXURE II. Table of corresponding values of u , $W(u)$, and $F(u)$

u	$W(u)$	$F(u)$	u	$W(u)$	$F(u)$	u	$W(u)$	$F(u)$
5	1.14(-3)	7.34(-2)	9(-2)	1.92	9.13(-1)	9(-4)	6.44	
4	3.79(-3)	8.99(-2)	8(-2)	2.03	9.56(-1)	8(-4)	6.55	
3	1.30(-2)	1.17(-1)	7(-2)	2.15	1.00	7(-4)	6.69	
2	4.89(-2)	1.57(-1)	6(-2)	2.30	1.06	6(-4)	6.84	
1	2.19(-1)	2.59(-1)	5(-2)	2.47	1.13	5(-4)	7.02	
			4(-2)	2.68	1.2	4(-4)	7.25	
9(-1)	2.60(-1)	2.76(-1)	3(-2)	2.96	1.32	3(-4)	7.53	
8(-1)	3.11(-1)	3.01(-1)	2(-2)	3.35	1.49	2(-4)	7.94	
7(-1)	3.74(-1)	3.27(-1)	1(-2)	4.04	1.77	1(-4)	8.63	$F(u) =$
6(-1)	4.54(-1)	3.60(-1)						$= \frac{W(u)}{230}$
5(-1)	5.60(-1)	4.01(-1)	9(-3)	4.14	1.92	9(-5)	8.74	
4(-1)	7.02(-1)	4.55(-1)	8(-3)	4.26	1.57	8(-5)	8.86	
3(-1)	9.06(-1)	5.32(-1)	7(-3)	4.39	1.92	7(-5)	8.99	
2(-1)	1.22	6.47(-1)	6(-3)	4.54	1.99	6(-5)	9.14	
1(-1)	1.82	8.74(-1)	5(-3)	4.73	2.07	5(-5)	9.33	
			4(-3)	4.95	2.16			
			3(-3)	5.23	2.28			
			2(-3)	5.64	2.46			
			1(-3)	6.33	2.75			

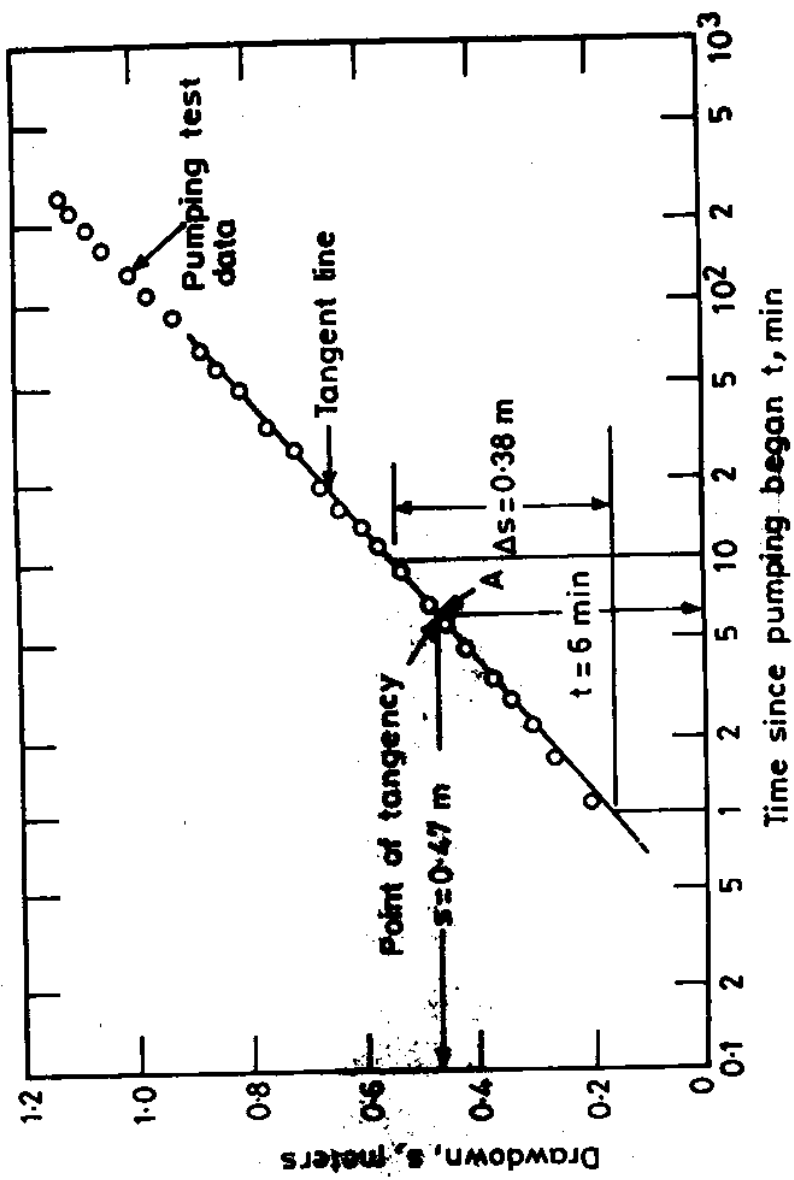


Fig. 3 Chow method for solution of the nonequilibrium equation

$$s = \frac{Q}{4\pi T} \left\{ -0.5772 - \ln u + u - \frac{u^2}{2.2} + \frac{u^3}{3.3} - \dots \right\}$$

$$\text{from } u = \frac{r^2 S}{4Tt}$$

It is seen that u decreases as the time of pumping increases, Thus, the terms beyond $\ln u$ in the series of the above equation become negligible. So for small value of u ($u < 0.01$) the drawdown can be expressed by the asymptote,

$$s = \frac{Q}{4\pi T} \left\{ -0.5772 - \ln \frac{r^2 S}{4Tt} \right\}$$

After rewriting and changing to decimal logarithms the above eq. reduces to,

$$s = \frac{2.30Q}{4\pi T} \log_{10} \left\{ \frac{2.25Tt}{r^2 S} \right\} \quad (2.10)$$

A plot of drawdown 's' versus the logarithm of t forms a straight line. This line is extended till it intercepts the time axis where $s=0$, so the interception point has the coordinates $s=0$ & $t = t_0$, substitution of these values in to eq.(2.10) gives

$$0 = \frac{2.30Q}{4\pi T} \log_{10} \left\{ \frac{2.25Tt_0}{r^2 S} \right\} \quad \text{and because}$$

$\frac{2.30Q}{4\pi T}$ can not equal to zero, so it implies that

$$\frac{2.25Tt_0}{r^2 S} = 1$$

$$S = \frac{2.25Tt_0}{r^2} \quad (2.11)$$

If $t/t_0 = 10$ and hence $\log t/t_0 = 1$, s can be replaced by Δs , i.e.; the drawdown difference per log cycle of time and it follows that

$$s = \frac{2.30Q}{4\pi T}$$

or,

$$T = \frac{2.30Q}{4\pi \Delta s} \quad (2.12)$$

Procedure (i)

Plot for one of the piezometers ($r=\text{constant}$) the values of ' s ' versus the corresponding time ' t ' on single logarithmic paper (t on logarithmic scale) and draw a straight line through the plotted points. Extend the straight line till it intercept the time axis where $s=0$, and read the value of ' t_0 ', (Fig.4).

Determine the slope of the straight line i.e.; the drawdown difference (Δs) per log cycle of time. Substitute the value of Q , & Δs in eq. (2.12) and solve for T with the known value of T calculate S from eq. (2.11).

Procedure (ii)

In this procedure a graph is plotted between ' s ' & ' r ' on a single logarithmic paper (r on logarithmic scale) (for $t=\text{constant}$) Again a straight line is fitted through the plotted points and extended till it intercepts the ' r ' axis where $s=0$. The interception point has the coordinates $s=0$ & $r=r_0$ ($r_0 =$ radius of influence at the chosen moment t) following the same line of reasoning as in procedure(i), the following eq. are derived.

$$S = \frac{2.25Tt}{r_0^2} \quad (2.13)$$

and,

$$T = \frac{2.30Q}{4\pi \Delta s} \quad (2.14)$$

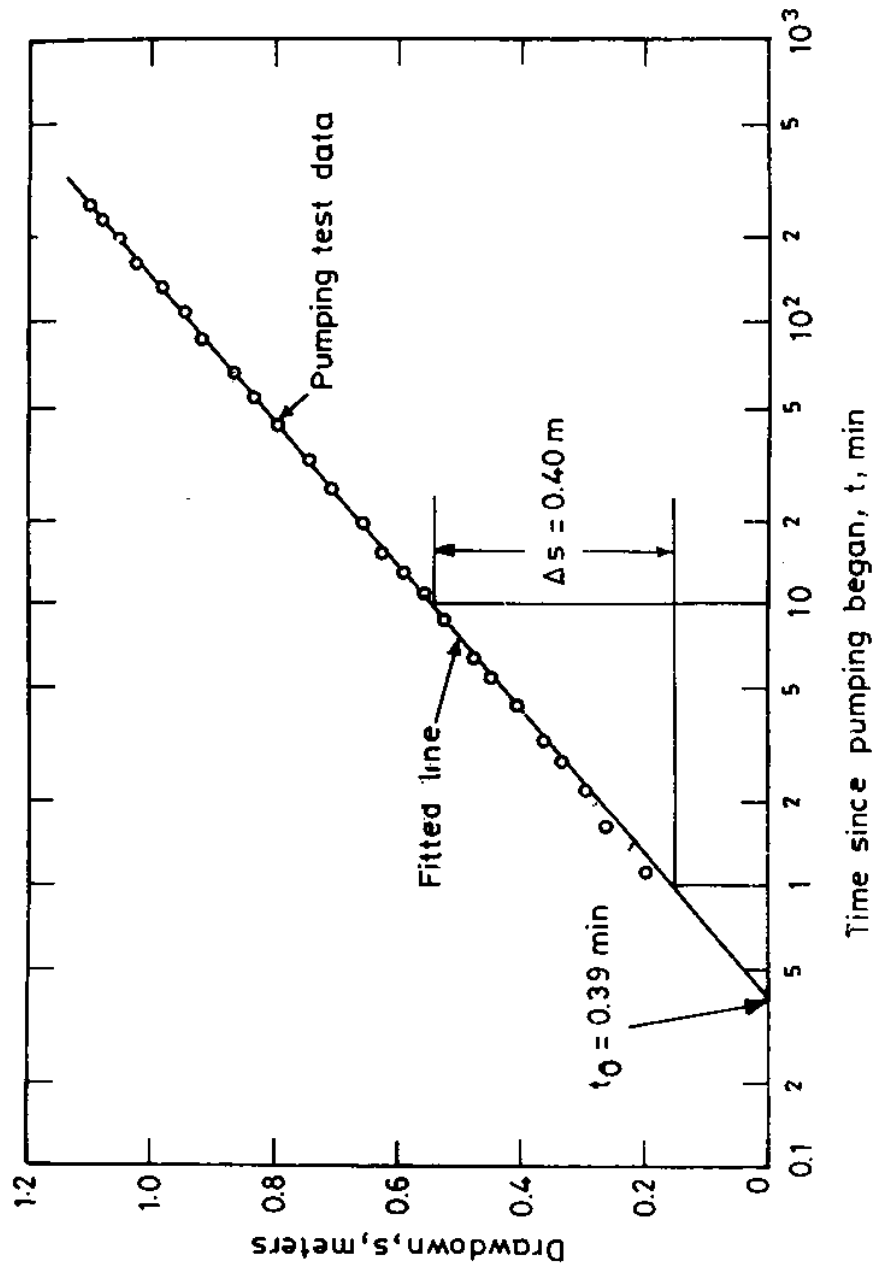


Fig. 4 Cooper - Jacob method for solution of the nonequilibrium equation

The value of r_0 & Δs are read from the graph (Fig.5), and with the eq. (2.13) and (2.14) the value of T & S can be calculated.

Procedure(iii):

If all the drawdown data of all the piezometers can be used in one graph on a single logarithmic scale then 's' is plotted versus t/r^2 (t/r^2 on logarithmic scale) A straight line is drawn through the plotted points and the intercept with the zero drawdown axis is determined. The coordinates of this point are $s=0$, & $t/r^2 = (t/r^2)_0$, following the same line of reasoning as in procedure (i) the following formulae are derived.

$$S = 2.25 T (t/r^2)_0 \quad (2.15)$$

and,

$$T = \frac{2.30Q}{4\pi\Delta s} \quad (2.16)$$

The numerical values of $(t/r^2)_0$ and Δs are determined from the graph (Fig.6), and T & S are calculated from the equations (2.16) & (2.15).

Assumptions for Jacob method are,

- (1) All the assumptions as for the Theis method and.
- (2) The value of u is small ($u < 0.01$), i.e.; r is small & t is large.

THEIS RECOVERY METHOD

After pumping has been shut down the water level will rise again to its original position. The rise of water level can be measured as residual drawdown s'' . The transmissivity can be calculated from the data obtained during recovery. Thus this method gives a check on the results of the analysis of the data obtained during the pumping period. Moreover, this method gives more accurate results than the pumping test method, because, there is no drawdown variation during recovery period as the rate of recharge 'Q' remains constant.

The residual drawdown, s'' , during the recovery period, is given by

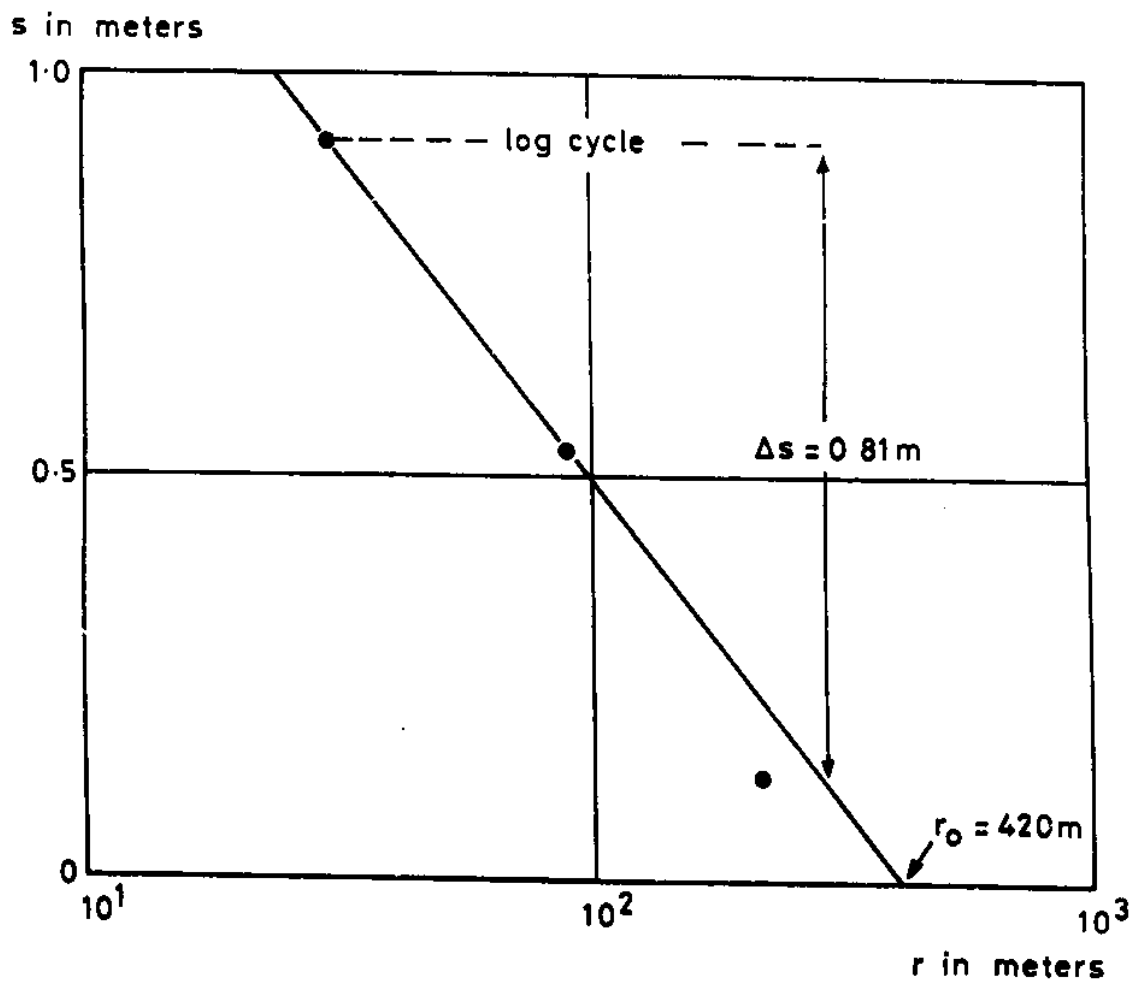


Fig. 5 Analysis of data from pumping test 'Oude Korendijk' ($t=140$ min) with the Jacob method, Procedure II

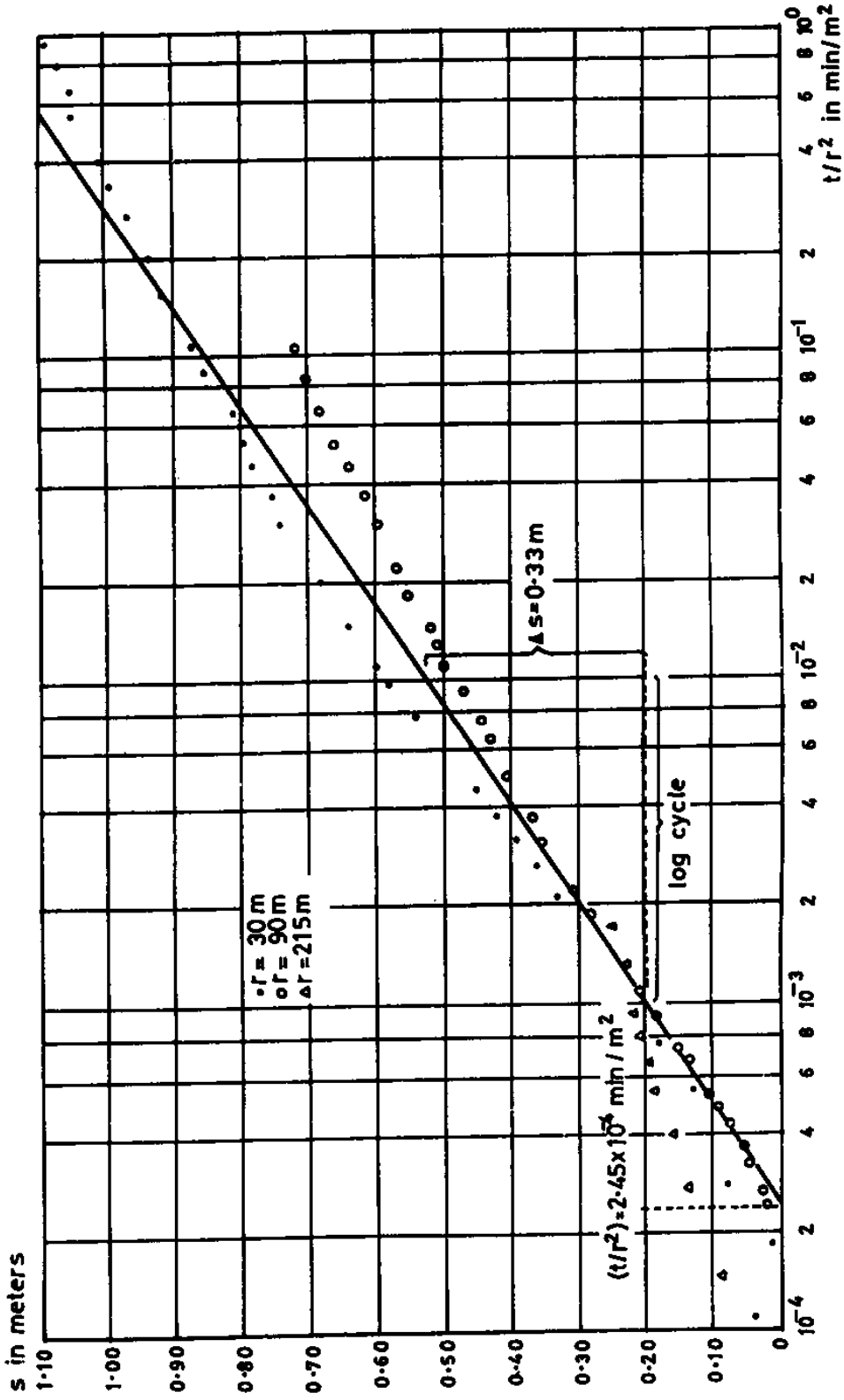


Fig. 6 — Analysis of data from pumping test 'Oude Korendijk' with the Jacob method, Procedure III

$$s'' = \frac{Q}{4\pi T} \left\{ \ln \frac{4Tt}{r^2 S} - \ln \frac{4Tt''}{r^2 S''} \right\} \quad (2.17)$$

where,

s'' is residual drawdown,

r is distance from pumped well to observation well or, if the pumped well itself is considered, $r=r_w$ (The effective radius of the pumped well),

S'' is coefficient of storage during recovery,

S is coefficient of storage during pumping,

t is time since pumping started,

t'' is time since pumping stopped,

Q is rate of discharge which is equal to rate of recharge.

Procedure

When S & S'' are constant and equal and $u = r^2 S / 4Tt''$ is sufficiently small, then eq. (2.17) can also be written as below.

$$s'' = \frac{2.30Q}{4\pi T} \log_{10} \left(\frac{t}{t''} \right) \quad (2.18)$$

For one of the piezometers or for the pumped well it self, S'' (residual drawdown) is plotted versus (t/t'') on a single logarithmic paper (t/t'' on logarithmic scale) and a straight line is fitted through the plotted points. The slope of this line is $(\Delta s'')$ equal to $2.30Q/4\pi T$, hence the value of $\Delta s''$, i.e.; the residual drawdown difference per log cycle of (t/t'') , can be read from the graph (Fig.7) and substitute in to the following eq. to find the value of T .

$$T = \frac{2.30Q}{4\pi \Delta s''} \quad (2.19)$$

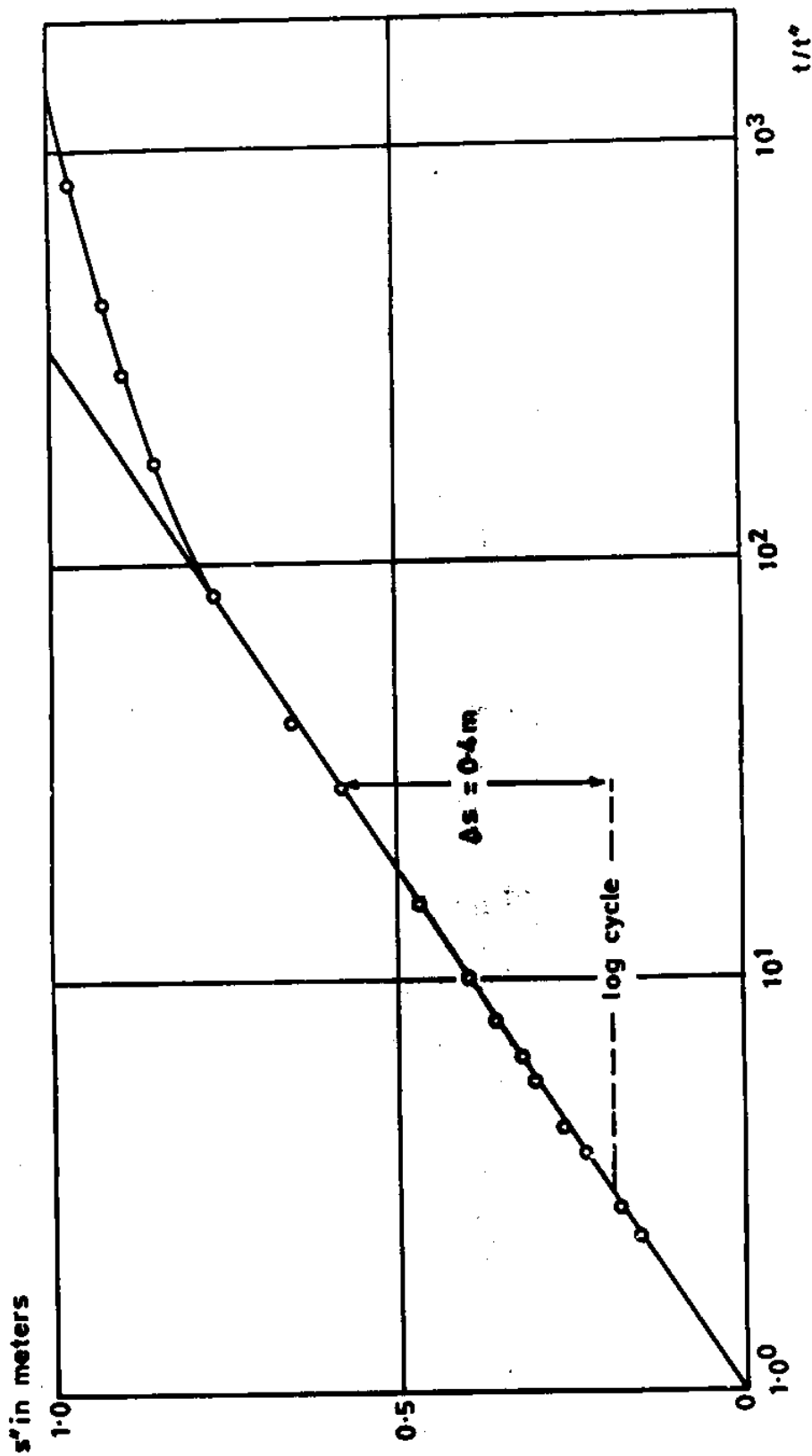


Fig.7 Analysis of synthetic recovery data from pumping test Oude Korendijk.
($r=30m$) with the Theis recovery method.

- (1) The value of 'S' can not be obtained with This method.
- (2) When S & S^{*} are constant but unequal then the straight line through the plotted points intercepts the time axis where s^{*}=0 at a point t/t^{*}=(t/t^{*})₀, and at this point eq. (2.17) becomes

$$0 = \frac{2.30Q}{4\pi I} \left\{ \log \left(\frac{t}{t^*} \right)_0 - \log \left(\frac{S}{S^*} \right) \right\}$$

Since, $\frac{2.30Q}{4\pi I}$ can not be equal to zero,

Thus $\log(t/t^*)_0 - \log(S/S^*)=0$

hence, $(t/t^*)_0 = S/S^*$

which determines the relative change of S and S^{*}.

3.0 COMPUTER BASED METHODS

Rushton(1978) used a numerical model for determining the transmissivity and the storage coefficient from abstraction well-water levels measured during both the abstraction and recovery phases. The technique is applied to a practical test. In addition to this, an ideal test is devised and the technique is used to give satisfactory estimates of the original parameters.

Smith and Piper(1978) presented an algorithm for the estimation of the aquifer parameters of transmissivity and storage coefficient from historic water level and recharge data . The estimated parameters can be used to develop a transient Ground Water model. The estimation of these parameters is in two parts. The transmissivities are estimated initially for periods of balanced recharge using the steady-flow equations. Using these transmissivities, the storage coefficients are estimated from the transient flow equations. No additional simplifications (such as a polynomial surface representation of transmissivity) are required, thereby aiding the hydrogeological interpretation of the results. The finite-difference method of successive over - relation is used for the numerical solution of the steady and transient flow equations. For the estimation of both transmissivity & storage coefficient a hybrid non linear optimization method is used to minimize the difference between the observed water levels and those predicted from the flow equations.

The method forms an important link between the more objective approach often applied to purely theoretical problems, and the adhoc trial and error procedures that are generally used to develop practical aquifer management models. The algorithm has been applied to the calibration of an aquifer model for the Tehran region of Iran. The results show that the mean error was reduced by a factor of three with respect to the initial parameter estimates.

Although parameters estimated solely from numerical methods will generally produce acceptable results that satisfy an objective error criterion on a sequence of historic data, these results might be physically unrealistic. Moreover, the construction of a convergence criterion based only on numerical results as suggested by Garay et al.(1976) implicitly assumes data in which the errors are insignificant, in most practical problems this is not the case.

Raner(1980) has given a program for a hand held programmable calculator for the analysis of pumping test data. The hand held calculator program enables the

geohydrologist to utilize computer capability at the test site to rapidly calculate transmissivity and storage coefficients of an aquifer without the need to construct curves or consult reference tables.

Subbir(1982) used two programs for the CASIO FX-502 programmable calculator for direct computation of transmissivity and storativity from time-drawdown and distance-drawdown data; using Jacob's modification of the Theis equation by the least squares method. The least squares method becomes very tedious for large samples, this is where programmable hand calculators can be useful. The programs also calculate drawdowns at various times and distances using the computed values of transmissivity and storativity. The advantage of the least squares method is the automatic objectivity, which is lacking in the type curve-matching or other graphical methods.

Dumble and Kelvin(1983) made use of microcomputer in the analysis of pumping test data. From the application of microcomputers, it becomes possible to rapidly generate and graphically compare theoretical time-drawdown curves with field data from pumping test in a confined aquifer, at a constant or varying rate for variety of T & S values and including boundary conditions. The ability to graphically display with field and theoretical data on a monitor screen removes the need for manual curve matching. This technique has the advantage over standard curve-matching methods in the sense that any solution can be tested over the complete range of field data.

Gupta and Joshi (1984) developed an algorithm for the Theis solution of pumping test data considering the standard graphical curve-matching procedure. The method is simple and it does not require initial estimates of transmissivity and storativity. As a measure of error in fitting, the integral square error is computed between the observed drawdown and drawdown calculated from the Theis equation. Also, root mean square deviation in drawdown is calculated. The integral square error for the drawdown should not be more than 3% for satisfactory results(Sharma et al,1969); otherwise, either one has questionable data or there exists a hydrologic situation which can not be represented by the Theis equation. By testing the algorithm with several synthetic data sets with varying magnitudes of error and varying distributions of error points in the data set, limitations of the applicability of the algorithm are pointed out. The estimates of the aquifer parameters obtained with the proposed algorithm for a typical field test data compare satisfactory with the estimates by the sensitivity approach developed by McElwee(1980). The algorithm is capable of identifying data with errors in observation or recording. The proposed algorithm identifies a group of

observations having the best match with the type curve. In this way, it provides objective estimates for the analysis of pumping test data using the Theis curve-matching procedure.

Rathod and Rushton(1984) described that how a numerical method of pumping test analysis, can be run on microcomputers. Full details of a program in BASIC and a test problem is provided. Provided sufficient care is taken, it becomes possible to carry out a pumping test analysis using numerical methods on microcomputers, however, the program is checked thoroughly, since the accuracy of computation varies for different microcomputer systems. The use of this numerical model is both for the analysis of pumping tests which are difficult to interpret using conventional methods, and for the prediction of the likely response due to extensive pumping from an aquifer.

Subhash, Kapoor and Goyal(1984) used Marquardt algorithm for estimating aquifer parameters from pump test data in nonleaky and leaky aquifers. from the study, it is observed that both in leaky and nonleaky aquifers the residual square error is the least in the case of Marquardt's algorithm. Further, it is also established that the method is not sensitive to initial estimates of parameters and the convergence is fast.

Hemker(1985) evolved a method of nonlinear regression analysis based on the principle of least squares, which can be applied to any flow system for which analytical expressions of the drawdown distribution are known. Marquardt's algorithm has been successfully implemented on a microcomputer. In view of the growing general interest in the application of microcomputers in ground water hydrology, a basic routine has been developed for estimating any number of aquifer parameters by the least squares method whose solutions is calculated by Marquardt's algorithm, using the singular value decomposition of the Jacobian matrix. The Robust computing method obtained can be applied to all kinds of pumping tests. Aquifer characteristics and their standard deviations are computed with optimal speed and accuracy. The basic routine presented in this paper is applicable to a large number of aquifer test problems, all of which may be solved by the appropriate drawdown formula which can be evaluated with sufficient accuracy.

Mukhopadhyay(1985) developed a new automated technique for the derivation of aquifer transmissivity and storage coefficient from pumping test data based on the Theis type curve-matching method. The Values of transmissivity and storage coefficient

for a non leaky confined aquifer are commonly obtained by manual curve-matching of pumping test data. Automatic matching of pumping test data with the Theis type curve is possible using the method of least squares, provided the Cooper-Jacob approximation of the well function $W(u)$ is considered. In this method A FORTRAN program was developed to carry out the calculations. This method gives unbiased estimates of T & S as long as the parameter $u < 0.05$. The program can eliminate data points that do not give a good fit.

Paschetto and McElwee(1985) presented a program written for an HP-41C, with two memory modules, up to 44 drawdown-time pairs which can be handled simultaneously. The 'best' T & S in the least squares sense is obtained. The complete Theis equation is used so there is no time or distance limitation. A feature of this program is the calculation of the rms error in the drawdowns. An error of several tenths of foot or more would indicate either poor data or a hydrologic situation that can not be represented by the Theis equation.

Sen(1986) evolved a method by which the slope between any two successive data points on a time-drawdown plot can be used for determining aquifer parameters in nonleaky and leaky aquifers. This method is known as "A type curve slope - matching method i.e.; S.M. method". This method yields values of transmissivity and storage coefficient which are in good agreement with the results of the classically known techniques. The important aspect of this method is its ability to determine a possible range in the values of T & S and to calculate confidence limits for average values of aquifer parameters. The method gives meaningful aquifer parameters estimates even for short duration pumping tests. The method is capable of analyzing erratic variations in the aquifer test data and avoids conventional type curve matching sequences of storativity and transmissivity estimates. Moreover, the method is not restrictive in its implementation of slopes which, in turn, is dependent on accurate measurements of drawdown with time. This method can be applied to any aquifer provided that the slopes of the appropriate type curve are known. This method does not require high speed digital computers. Rather, a simple hand calculator is adequate for all the calculations involved.

Swamee and Ojha(1990) developed a high accuracy expression for the well function valid for the entire range of its argument. The present practice of aquifer parameter determination uses approximations of well function valid for small range of arguments only. In the absence of a full-range equation for the well function, it is

not possible to utilize the entire set of pump test data. This practice of parameter estimation uses least square method which gives undue importance to large errors. In This paper criteria function capable of ignoring large observational errors has been developed for the replacement of the least square method. Minimizing the cumulative criteria function, the aquifer parameters can be estimated with a high degree of accuracy.

4.0 FEW RECENT GRAPHICAL METHODS

Griffiths and Ife (1984) developed a new method which employs a transparent overlay that permit direct reading of transmissivity and storage coefficient without numerical calculation for any set of time-drawdown readings. Distance-drawdown readings can also be analyzed in the same way, and the overlay can be used to determine the time-drawdown curve for a known set of aquifer parameters. A single time-drawdown point can produce one aquifer characteristic if an assumption of the other characteristic is made.

The overlay method assists in the understanding of the way in which the various parameters of the Theis equation interrelate and as such can be used in the design and interpretation of aquifer tests. The graphical technique proposed enables various analyses of pumping test data to be carried out in the field as test proceed.

Goyal(1984) presented a new approach for the analysis of aquifer test data for confined nonleaky aquifers using an implicit function from intersecting graphs, which are drawn for different drawdown observations (Fig.8 & 9). The proposed method is simple and requires a small number of observations, thus cutting short the duration for which the aquifer test should be conducted. In this method, no curve-matching is involved and the results are reasonably accurate. The method has been successfully applied to actual aquifer test data from a confined nonleaky aquifer.

Singh and Ram(1996) evolved a straight line method valid for full range of well function argument. The Jacob straight line method and the Theis curve - matching method are most widely used for the identification of aquifer parameters from pump test data. Jacob method is not applicable when the argument of the well function is greater than 0.01 (i.e.; $u > 0.01$). In Theis method, considerable subjectivity is involved in curve - matching. Both Jacob method and Theis method give less weight to the early drawdown data. This method is reliably accurate for aquifer parameter identification from short duration pump test data(Fig.10).

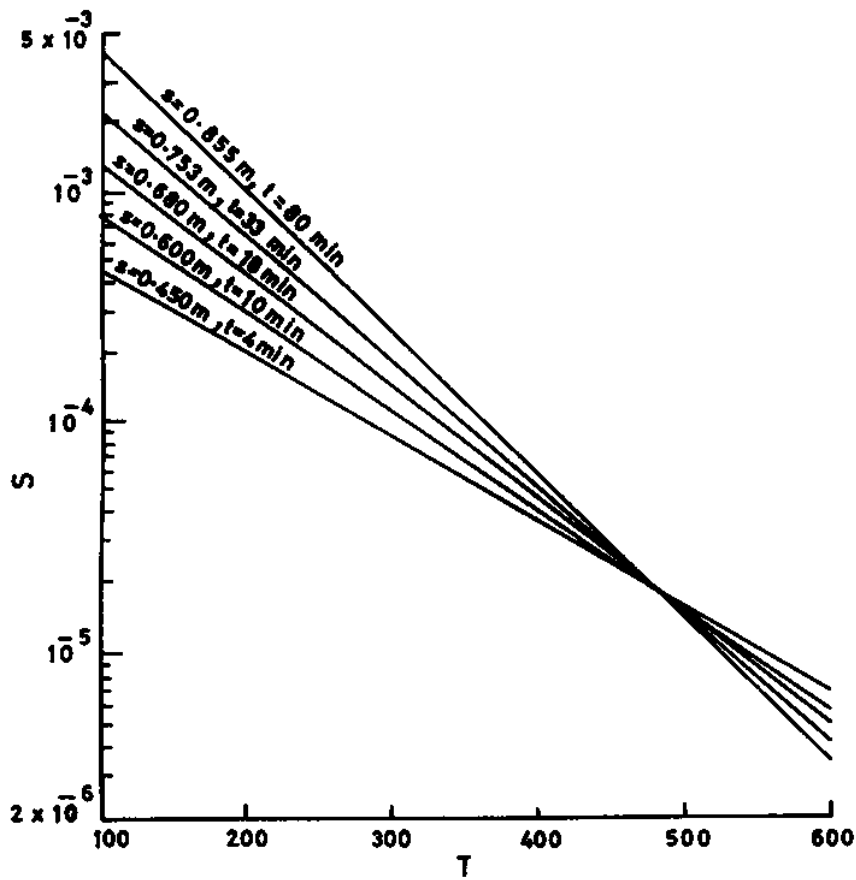


FIG.8-Graphs of S and T Indicated Drawdowns ($r=30$ m)

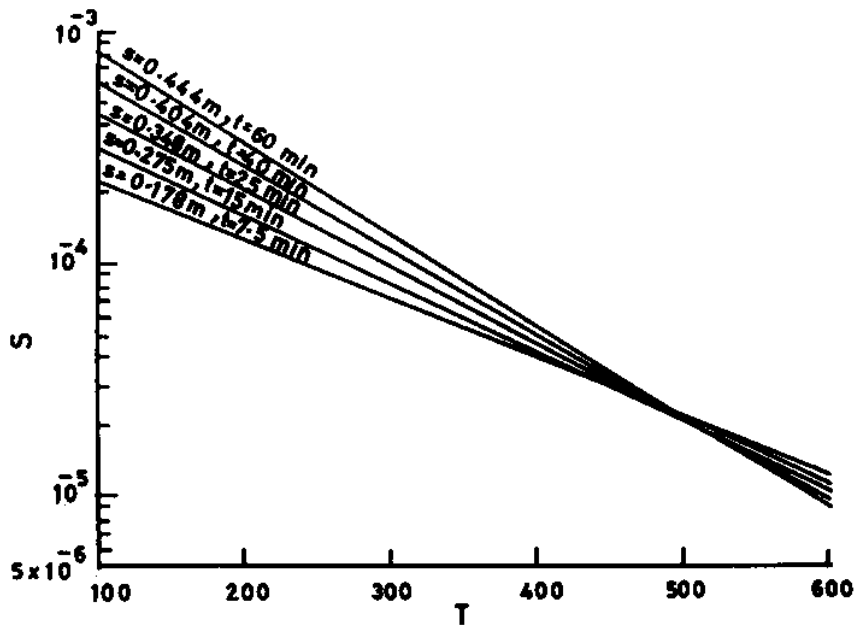


FIG.9-Graphs of S and T for Indicated Drawdowns ($r=90$ m)

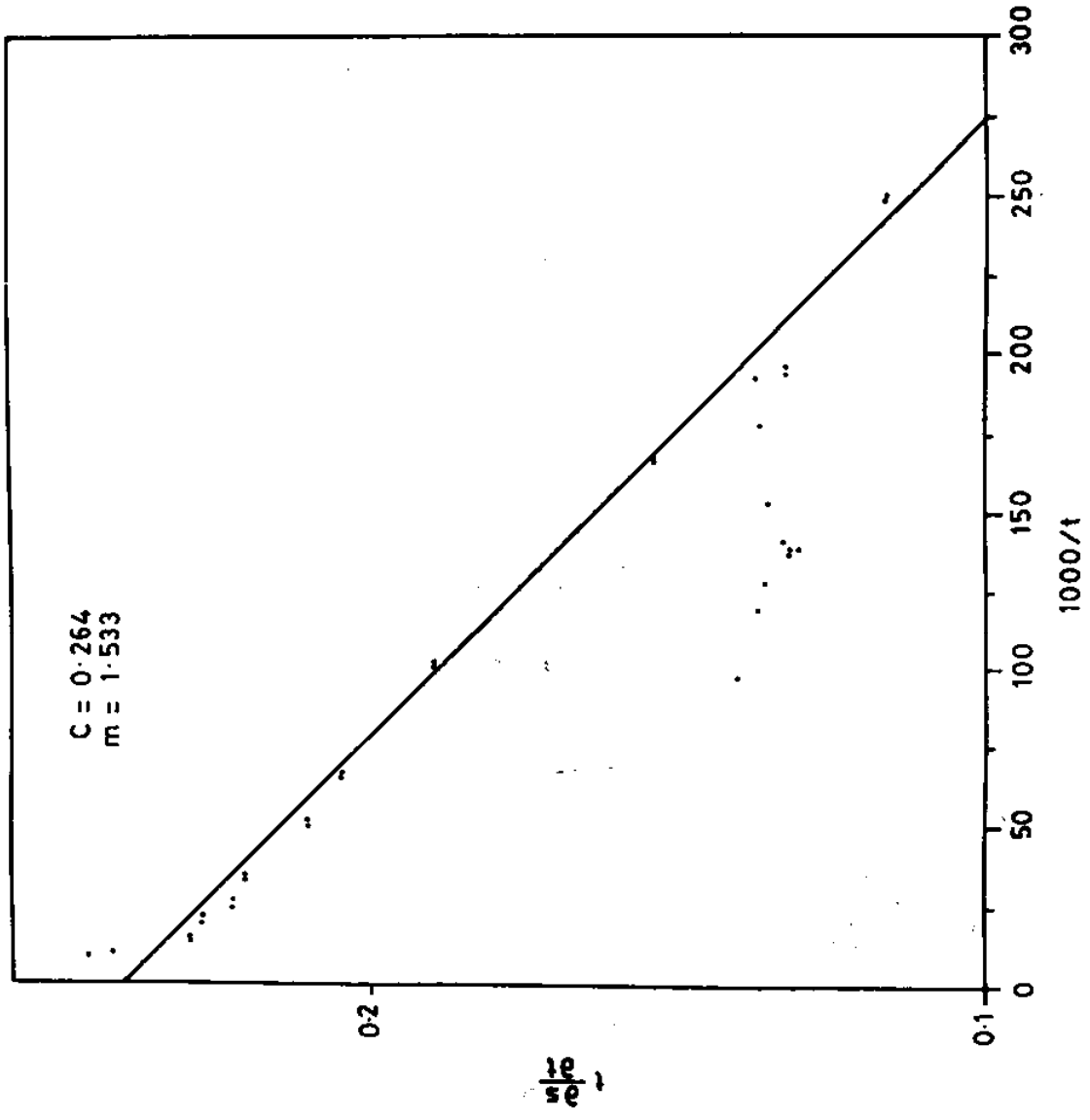


FIG.10 SEMI-LOG GRAPH FOR $r = 99.9$ m

Franke(1987) developed a new set of type curves (Fig.11 & 12), based on three dimensionless parameters, sT/Q , Qt/sr^2 , and S , that can be employed to analyze aquifer test data by a curve-matching procedure used at present with two dimensionless variables.

The possible advantages of the proposed new type curves are as follows.(1) The concave shape of the type curves along the X-axis and particularly their point of vertical tangency permits curve-matching with a minimum of ambiguity. Further more, the possibility of gross error in the curve-matching procedure is reduced because both type curves and the plot of field data use the same dimensionless parameters on the horizontal axis (X-axis), so that the process of curve-matching permits the movements of the field curve only parallel to the vertical axis (Y-axis). (2) The shape of the proposed type curves provided a focus for aquifer test design-that is, given very approximate estimates for the field parameters (T & S). The investigator can target a specific range for the field - measured parameters for which curve-matching will give best possible results.

The specified features of the proposed type curves listed above, may prove to be useful in analyzing field situation (with corresponding data sets) that obviously deviate from the assumptions of the Theis solution.

5.0 CONCLUDING REMARK

In this report, the methods of evaluation of confined aquifer parameters have been reviewed. All the methods reviewed in this report has been divided in to three groups such as (1) Traditional methods, (2) Computer based methods, and (3) New graphical methods. In traditional methods, the methods reviewed are, Thiem method, Theis method, Chow method, Cooper and Jacob method and Theis recovery method.

Under the head computer based methods, the methods reviewed are, (1) New automated techniques for the derivation of aquifer parameters, (2) Application of microcomputer in the analysis of pump test data, (3) Programmable hand calculator programs for pumping test analysis by least squares method, (4) Methods of non linear regression analysis based on the principles of least squares, (5) Analysis of pumping test data using marquardt algorithm, (6) Non linear optimization method for the estimation of aquifer parameters.

Under the head new graphical methods, the methods reviewed are, (1) Alternate procedure for analyzing aquifer tests using the Theis nonequilibrium solution, (2) A simplified graphical solution of the Theis equation, (3) New approach for analysis of aquifer test data, (4) A new graphical method for identification of aquifer parameters.

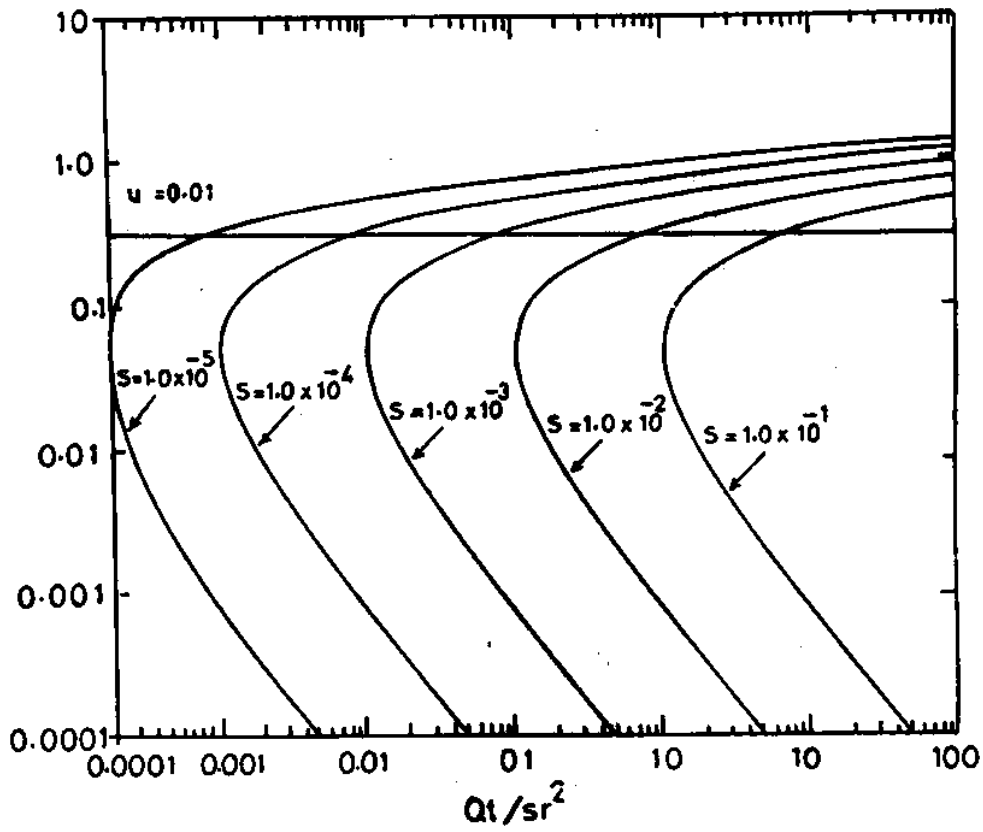


FIG.11. PLOT OF TYPE CURVES WITH PARAMETERS sT/Q VERSUS Qt/sr^2 FOR SELECTED VALUES OF $S = \text{CONSTANT}$

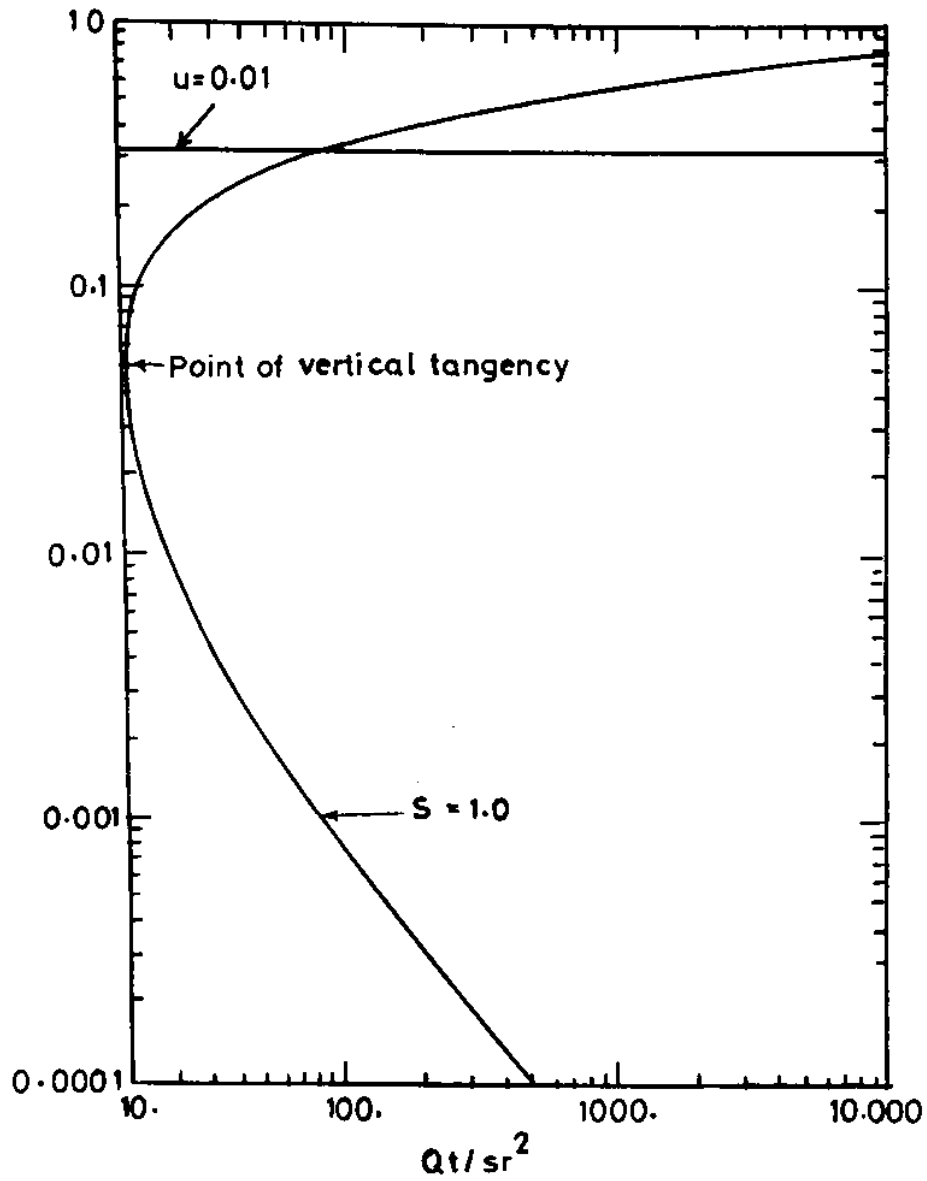


FIG.12. PLOT OF TYPE CURVE WITH PARAMETERS sT/Q VERSUS Qt/sr^2

REFERENCES

1. Dumble, J.P., and Kevin, T.C. (1983). The application of a microcomputer in the analysis of pumping test data in confined aquifer. *Ground Water* Vol.21, No.1, pp.79-83.
2. Franke, O.L. (1987). An alternate procedure for analyzing aquifer tests using the Theis Nonequilibrium solution. *Ground Water*, Vol.25, No.3, PP.314-320.
3. Garay, H.L., Haines, Y.Y. and Das, P. (1976). Distributed parameter identification of ground water systems by nonlinear estimation, *Hydrology*, vol.30, no.(1/2), pp. 47-61.
4. Goyal, S.K. (1984). New approach for analysis of aquifer test data, *Irrig. & Drain. Div.*, Vol.110, no.4, pp. 388-392.
5. Griffiths, W T., and Ife, D. (1984). A simplified graphical solution of the Theis equation. *Hydrology*, Vol.73, PP. 177-186.
6. Gupta, D.A. and Joshi, S.G. (1984). Algorithm for Theis solution. *Ground Water*, Vol.22, No.2, PP.199-206.
7. Hemker, C.J. (1985). A general purpose microcomputer aquifer test evaluation technique. *Ground Water*, Vol.23, No.2, pp.247-253.
8. McElwee, C.D. (1980). Theis parameter evaluation from pumping tests by sensitivity analysis, *Ground Water*, Vol.18, no.1, pp. 56-60.
9. Mukhopadhyay, A. (1985). Automated derivation of parameters in a nonleaky confined aquifer with transient flow. *Ground Water*, Vol.23, No.6, pp.806-811.
10. Paschetto, J. and McElwee, C. D. (1982). Hand calculator program for evaluating Theis parameters from a pumping test. *Ground Water*, Vol.20, No.5, pp.551-555.
11. Rathod, K.S. and Rushton, K.R. (1984). Numerical method of pumping test analysis using microcomputers. *Ground Water*, Vol.22, No.5, PP.602-608.
12. Rayner, F.A. (1980). Pumping test analysis with a hand held calculator. *Ground Water*, Vol.18, No.6, PP.562-568.
13. Rushton, K.R. (1978). Estimating transmissivity and storage coefficient from abstraction well data. *Ground Water*, Vol.16, PP.81-85.
14. Sarma, P.B.S., Delleur, J.W., and Rao, A.R. (1969). An evaluation of rain fall run off models for small urbanized water sheds and the effect of urbanization on runoff. A program in urban hydrology. Part 11, Purdue University, Water Resources Research Centre, Lafayette, Indiana.
15. Sayed, S.A.S. (1982). Programmable hand calculator programs for pumping test analysis by least squares method using Jacob's modification of Theis equation. *Ground Water*, Vol.20, No.2, pp.156-161.
16. Sen, Z. (1986). Determination of aquifer parameters by the slope-matching method. *Ground Water*, Vol.24, No.2, PP.217-223.

17. Singh, S.K. and Ram, S. (1996). A new graphical method for identification of aquifer parameters. All India Seminar on "Hydraulic Engineering" Jan.96, I.E.(I), Nagpur
18. Smith, P.J., and Piper, B.S. (1978). A non linear optimization method for the estimation of aquifer parameters. Hydrology, Vol.39, PP.255-271.
19. Subhash, C., Kapoor, P.N. and Goyal, S.K. (1981). Analysis of pumping test data using Marquardt algorithm. Ground Water, Vol.19, No.3, PP.275-278.
20. Swamee, P.K. and Ojha, C.S.P. (1990). Pump test analysis of confined aquifer, Irrig. & Drain. Div., Vol.116, no.1, pp 99-106.