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One-Dimensional Modelling of Branched Free Surface Flow

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ABSTRACT

Occurrence of branched flow situation is quite common. Flow diversion is one of the techniques used to reduce flooding of a downstream city. This flow may rejoin the stream taking a circuitry route. Sometime, river length is reduced by means of cut-off channel as an channel improvement scheme. The diverted flow may take lower velocity than the main. On the other hand the flow in the cut-off diversion and rejoining at a down stream point. Flow computations in such situations need special attention as the computed flow depths need to satisfy certain conditions at the junctions. Therefore a mathematical model capable of computing the flow depth and the discharge in each of the branched is developed using gradually varied flow equations. The model is the first step towards modeling complex situations like one encounters in braided river.

This model can handle three flow situations viz. (I) temporary flow diversion; (ii) a cut-off channel and (iii) a single river island. Computational capability of the model is increased to handle irregular cross sections of a river. Parametric studies have been carried out using this model in order to find out the effect of the diversion channel width on the discharge it can carry for different values of bed roughness and bed slope. A relationship is established between the divided flow, width, slope and roughness coefficient in a dimensionless form for each of the flow situation using least square methods. These equations are derived for rectangular channel. However it is found that approximation to irregular geometry can be applied and the derived equations can be employed with confidence.

The model development and the results discussed in this report are for relatively simpler cases compared to the flow situations normally encountered. Suggestions are also given to improve this model further.

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1.0 INTRODUCTION

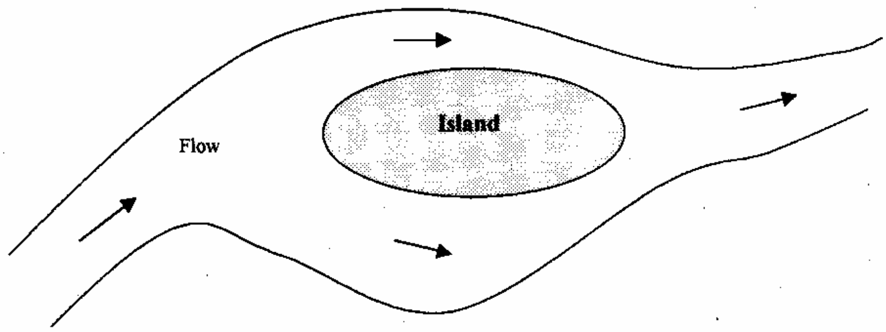
1.1 GENERAL

Free surface flows are defined as the flows where one or more of its boundaries is not physically constrained but can adjust itself to conform to the flow condition. Mathematically, free surface flows occur in a deformable solution region whereby the shape and size of the region is part of the solution (Liggett, 1993). As used in this work, the term free surface flow is more physical satisfying the first definition and thus, the strict mathematical definition of free surface flow is violated.

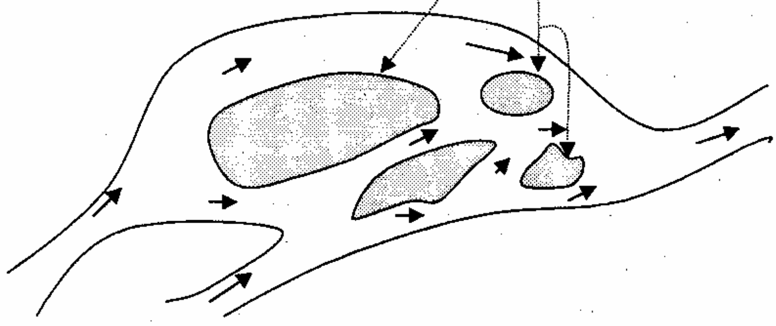
Branched free surface flows are very common in natural and artificial systems and may take place when the flow is divided into two or more séparate channels. Tributaries flow into and distributaries flow away from the main river. Deltas are formed as streams deposit bed load and coarser suspended sediments where carrying capacity of the streams are reduced. The stream gets braided if the bank material is easily eroded, if the sediment load is composed in large part of sands and gravels moving as bed load , and if the dunes that are formed are large (Petersen 1986). This gives rise to a very complex flow network. Branched flow may occur due to natural islands in a river. The irrigation system is always associated with branching due to the networking consisting of main canals, branch canals, minors and distributaries. The water distribution and the waste water disposal systems are also with branching and may have free surface flows. Diversion channels are often constructed for flood control purposes. Prior to a dam construction, provision of a diversion tunnel is a common practice. Cutoff channels are either man-made or natural and are useful to protect the banks of a highly meandering river.

Branched flows can be either looped or non-looped. In a looped flow, the branches join again at a downstream point(Fig.1) whereas the branches do not meet in an unlooped flow (Fig.2). Looped flows can be (i)single looped and (ii)multi-looped. The single looped flow is popularly known as simple island type flow (Fig.1a) and the multi-looped channel as multiple island type flow (Fig.1b). The non-looped flows are also known as tree-type branching. Whether looped or non-looped, it is advantageous to represent the flow situation by a network of nodes and links (Fig.3). Channel junctions can be divided into two categories - (i)point type junction and (ii) storage type junction (Yen 1979). For junctions with insignificant storage capacity, the junction can be considered as a point type junction. It is assumed to be represented by a single confluence point without storage. The net discharge into the junction is, therefore, zero at all times. The reservoir type of junction is assumed to behave like a reservoir with a horizontal water surface. It is capable of adsorbing and dissipating all the kinetic energy of the flow. The net discharge into the junction is equal to the time rate of change of storage in the junction.

The physical processes involved in a branched free surface flow are highly complex. The flow is unsteady, three-dimensional and turbulent. Transverse and bending flows are very prominent near the junction. If the branch channel is highly curved, the flow away from the junction may also have transverse components. A discontinuity in the water surface is possible due to a bore/shock. Inclusion of either lateral inflows/outflows or a porous media makes the flow spatially varied. The process is even more complex when the channel bed is mobile where dynamics of sediment plays a major role. It includes the processes of bank erosion and aggradation/degradation of channel bed. Transport of pollution may also take place. The flow



(a)Single island type flow



(b)Multiple island type flow

Fig. 1 Looped branching

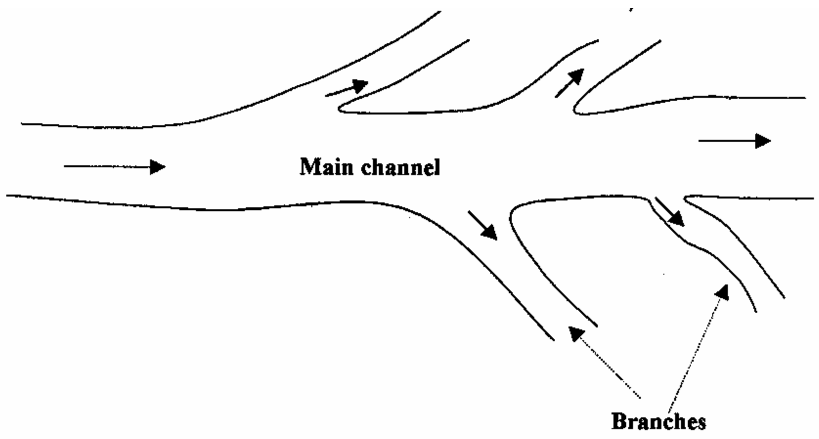


Fig. 2 Non-looped branching

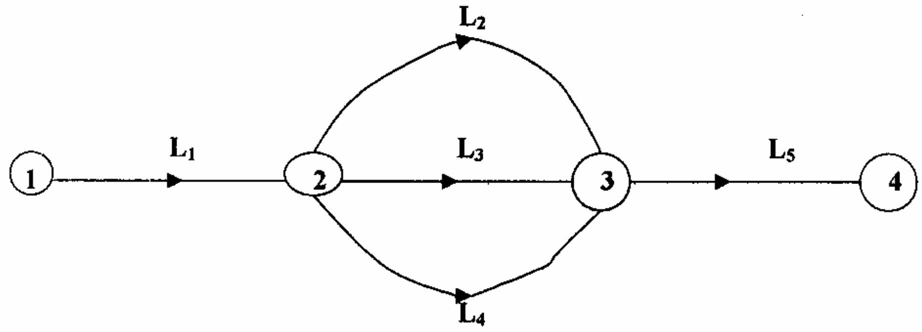


Fig. 3 Branched flow (Link-Node representation)

patterns are governed by the approaching flow, geometry of branches and the bed conditions.

Accurate analysis of flow variables in a branched free surface flow is essential in many engineering applications. Estimation of the free surface elevation helps in the determination of the effect of hydraulic structures on upstream and downstream channels, estimation of flood zone and determination of the safe and optimum operation of control structures. The design of diversion channels and cutoffs greatly depend on the quantity of flow to be passed through them. It also helps in the estimation of sediment and pollution transport.

The analysis of branched free surface flows can be done either experimentally, analytically or numerically. Although, experimental studies provides a better understanding of the problem, the time and cost required for such studies are very high. The scale effects are also to be considered. The analytical studies are limited to highly idealized cases. The numerical methods are preferred due to the availability of efficient numerical techniques and high speed computers. However, the numerical methods are also not free from various types of numerical errors and imposition of exact boundary conditions, sometimes, becomes difficult.

In the following section, literature review on flows in branched channels is presented. This includes the experimental, analytical and numerical studies on the above flows. Flows in both looped and non-looped open channels are considered for this purpose.

1.2 REVIEW OF LITERATURE

First detailed experimental studies of branched free surface flows was conducted by Taylor(1944). The branch off-take was at right angles to the main channel. Based on the experimental results, Taylor proposed a graphical solution that included a trial and error procedure to compute. He validated his procedure for field data. In the analytical studies by Milne-Thomson(1949) conformal transformation was used and equal flow depth in all the branches was assumed. Stoker(1957) proposed a computational procedure for the unsteady free surface flow in a channel junction. The experimental study by Grace and Priest(1958) was for different branch angles and different width ratios. He classified the branched flows depending on the Froud number of the flow.

Studies by Krishnappa and Seetharamiah (1963), Pattabhiramiah and Rajaratnam (1960) and Rajaratnam(1967) were based on the assumption of subcritical flow in the main horizontal channel and supercritical flow in the sloped branch channel. The branched flow was calculated using the weir formula depending on the flow head. Law and Reynolds(1966) Studied the branched flow with analytical and experimental studies where the width ratio is kept equal to one and dependence of the discharge ratios on the Froud number is established.

Hager(1983) derived loss coefficients for flow through branches by neglecting the transverse variations perpendicular to the channel flow. Li et al.(1983) used a difference method of flow in branched channel. Ramamurthy and Satish(1988) presented a theoretical model for division of flow in a short branched channel for various width ratios. The principles of momentum, energy and continuity are used in the analysis. Studies assuming supercritical flow in channel junctions were presented by Bowers(1950) and Hager(1989).

Chaudhry and Schulte(1986) used a Finite-Difference technique for steady flows in a channel with two parallel branches. Schulte and Chaudhry(1988) extended the above study for multi-island type flows. They used the simultaneous procedure. Satish et al.(1987) studied the pressure recovery in branched free surface flows that are spatially varied i.e. the discharge varies with space. Hager(1989) studied the transitions from subcritical to supercritical flow condition. He assumed rectangular channels with equal cross sections. Ramamurthy et al.(1990) obtained an estimate of the discharge ratio in terms of the Froud numbers in the main channel up- and down-stream of the junction. Choi and Molinas(1993) used a simultaneous solution algorithm using the double sweep method for the channel networks. Christodoulou(1993) studied the incipient hydraulic jump at channel junctions. The three-dimensional flow structure with sediments at an open channel diversion was investigated by Neary and Odgaard(1993). Dependency of the flow pattern on the roughness of channel bed and ratio of diversion flow velocity to main flow velocity was established. Nguyen and Kawano(1995) used a double sweep algorithm for dynamic wave flood routing in non-looped open channel networks. Biron et al.(1996) studied the effects of bed discordance on flow dynamics at open channel confluence with laboratory flume data.

Naidu et al.(1997) used an efficient computational procedure for the gradually varied flow surface computation in tree type channel networks. This study is for a steady state flow. The fourth order Runge-Kutta method was used to solve the ordinary differential equation and a more efficient shooting technique is used to connect the solutions. Gurram et al.(1997) presented the subcritical junction flow with free surfaces using a hydraulic model study with angles of 30, 60 and 90 degrees.

The above paragraphs show that most of the studies are for non-looped channels. Also, in many cases the branch is at right angles to the main channel. In most of the cited works, the interest is to know the flow pattern at the junction. In the present work, however, the main concern is the flow diversion. Thus, the overall effect rather than the local effect due to branching is considered here. In this work, there is an attempt to study various parameters affecting the flow in a single island type of branching.

1.3 OBJECTIVES AND ORGANISATION

A single island type of flow is considered through out this work. The channel bed and bank are assumed rigid and therefore, no equations for sediment flow is used. All flows in this study are assumed to be subcritical. The junctions are of point junction type and are fixed. The branch angles and the curvatures are such that transverse flows are unimportant and the flow in all the channels are assumed to be one-dimensional. All the channels including branch channels are of irregular cross-sections. However, wide rectangular cross-section is assumed in the parametric studies.

The main objectives of this study are;

(1)to compute the flow division and the water surface profile in all the branches assuming gradually varied steady flow,

(2) to study the effects of different parameters (bed roughness, channel width, bed slope) on the flow division in the branches, and,

(3) to compute the flow for a case with highly irregular areas of cross-sections.

The presentation of this work is as per the following organisation;

Equations governing the flow are presented in the second section and their solution strategy is given in the third section. Section four presents various results obtained by the present work. Important conclusions and recommendations for future study are presented in the last section.

2.0 GOVERNING EQUATIONS

2.1 GENERAL

The governing equations for the free surface flows are the mass, momentum and energy conservation laws. Except for the velocity head and momentum coefficients, both momentum and energy equations are equivalent (Cunge et al. 1980). The above statement is valid for continuous flow depth and velocity i.e. if there is no discontinuity such as a bore or a hydraulic jump. The continuity equation and the momentum equations (Navier-Stokes equations) in three-dimensional form completely describe the free surface flow on a rigid bed. It is common in civil engineering practices that the three-dimensional equations are converted to two-dimensional equations by depth averaging i.e. assuming a uniform velocity distribution along the vertical plane. The two-dimensional equations can again be simplified to one-dimensional equations when the variations in the transverse direction are neglected. The governing equations for the steady state flow can be obtained by neglecting the transient terms used in the unsteady flow equations. The partial differential equations are converted to ordinary differential equation. Any flow parameter is only a function of space. Thus, important results can be obtained as the solution becomes easier.

In the following section, governing equations for steady flow are presented, the assumptions and their validity for a single island type flow are also discussed.

2.2 STEADY FLOW EQUATIONS

CONTINUITY EQUATION:

$$Q = Const. \quad (1)$$

MOMENTUM EQUATION:

$$\frac{\partial y}{\partial x} = \frac{S_b - S_f}{1 - F_r . F_r} \tag{2}$$

In the above equations, Q is the flow discharge, y is the flow depth, S_b is the bed slope, S_f is the friction slope and F_r is the Froud number. The Froud number is given by,

$$F_r = \frac{Q}{A} \frac{\sqrt{T}}{\sqrt{gA}} \tag{3}$$

where, g is the acceleration due to gravity. T and A are the top width and area of cross-section, respectively, for the given flow depth, y.

The friction slope is computed assuming a uniform flow. Therefore, S_f is calculated from Manning equation by,

$$S_f = (Q^2 n^2 P^{4/3}) / (A^{10/3}) \tag{4}$$

where, P is the wetted perimeter and n is the Manning's roughness coefficient. Although n is in general a complicated function of flow depth, bottom roughness, slope, discharge and bed forms, a constant value is assumed through out this study. However, variation of n can be easily incorporated because explicit numerical method is used for the solution of the governing equations.

Above governing equations (Eqs. 1 and 2) use following assumptions.

1. The vertical pressure distribution is hydrostatic.

2. The channel bottom slope is small.

3. The flow velocity over the entire channel cross section is uniform.

4. The channel is prismatic.

Eq.2 can also be obtained from the energy equation. It is generally known as gradually varied flow equation (GVF).

2.3 EQUATIONS AT THE JUNCTION

The sections near the junction should satisfy the conservation of quantity and conservation of energy.

CONTINUITY EQUATION:

$$\Sigma Q_i - \Sigma Q_o = 0 \qquad (5)$$

In the above equation, Q_i and Q_o represent all inflow and outflow discharges respectively. The junction is assumed to be point type.

ENERGY EQUATION:

The energy equation between two sections is;

$$y_1 + z_1 + \frac{Q_1.Q_1}{2.A_1.A_1.g} - (y_2 + z_2 + \frac{Q_2.Q_2}{2.A_2.A_2.g} + E_l) = 0 \qquad (6)$$

where, z is the bed elevation and E_l is the energy loss. Subscripts *1* and *2* indicate the sections *1* and *2* respectively.

In the present section, governing equations for steady flow condition have been presented. The equations for the junction have also been presented. The numerical method to solve these equations is described in the next section.

3.0 NUMERICAL SOLUTION

3.1 GENERAL

The governing equations presented in the previous section are to be solved numerically. The main equation is the gradually varied flow equation (Eq. 2). This is an ordinary differential equation and it is solved to obtain the water surface profile. Besides, the continuity and the energy equations at a junction are to be satisfied. In this section, a mathematical model to solve the equations for a island type flow is presented. The simple island type flow is shown schematically in Fig.4.

The solution of the GVF equation (Eq.2) can be obtained by the numerical integration methods for ordinary differential equations such as Standard step method, Direct step method, Euler method, Modified Euler method, Standard Fourth-Order Runge-Kutta Method (Subramanya, 1985, Chaudhry 1993). The mathematical model used in the present work is presented below.

3.2 MATHEMATICAL MODEL

3.2.1 TERMS USED

The system consists of a channel dividing in two branches and then joining at a downstream site as shown in Fig. 4. N is the channel number and has a value 1,2,3 or 4 for the flow situations considered. The computational nodes in each channel are numbered from 1 to I_{NMAX}. Thus, for channel 2, the nodes will be from 1 to I_{2MAX}. I_{NMAX} depends on D_N, the number of divisions in the channel N. The length, bed slope, and roughness in any channel N are represented by L_N, S_{bN}, and n_N respectively. The two junctions are designated as J_1 and J_2. The flow depth $y(I_N)$ represents the value for channel N at node I. Discharge is constant for a given channel (following Eq.1). Therefore, discharge in the channel N is represented by Q_N. For a given channel

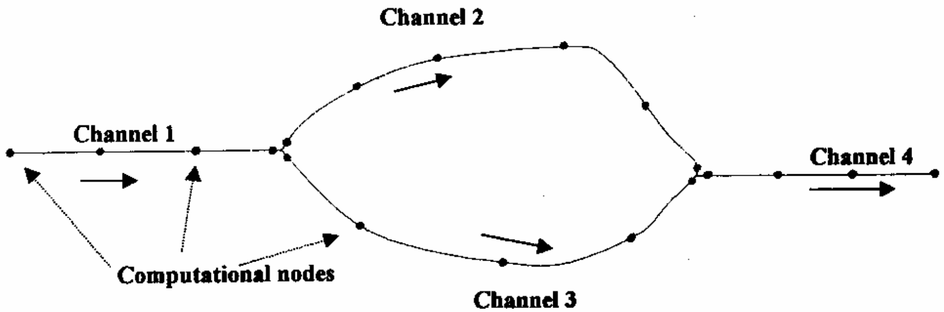


Fig. 4 A single island with computational nodes

(N), at a given section (I_N), the cross-section is represented by discrete values of area, $A_m(I_N)$, top width, $T_m(I_N)$ and perimeter, $P_m(I_N)$ as a function of height, $h_m(I_N)$, where, m is the number of discrete values.

3.2.2 SOLUTION STRATEGY

For a given discharge, Q_u, at the upstream of channel 1 and the flow depth, y_d, at the downstream of channel 4, the aim is to compute the flow depths in all the channels and the discharges in the branch channels (channels 2 and 3) as shown in Fig.4. The geometry and roughness in all the channels are known. The computational steps are shown in the flow chart(Refer Fig.5).

STEP 1: Setup the roughness coefficients and geometry for all the channels including the branch channels. The grids are set. Q_u and y_d are also read as input values.

STEP 2: Knowing the discharge, $Q_4(=Q_1)$ and depth of flow, $y(I_{4MAX})$, the GVF equation (Eq.2) is solved to compute depth of flow in channel 4, $y(I_4)$.

STEP 3: A guess for the division of discharges is applied. Q_2 is assumed and then Q_3 is computed by

$$Q_3=Q_1-Q_2 \qquad (7)$$

STEP 4: The energy equation (Eq.6) is solved for junction *2* to compute depth of flow at the downstream end of channels 2, $y(I_{2MAX})$ and channel 3, $y(I_{3MAX})$, respectively.

STEP 5: The GVF equation (Eq.2) is solved for channels *2* and *3* to compute flow depths in channel 2, $y(I_2)$ and channel 3, $y(I_3)$, respectively.

STEP 6: The energy equation is applied to junction *1* to get two values of depth of flow at downstream end of channel 1, $y(I_{1MAX})$ separately from computations through channel 2 as well as channel 3.

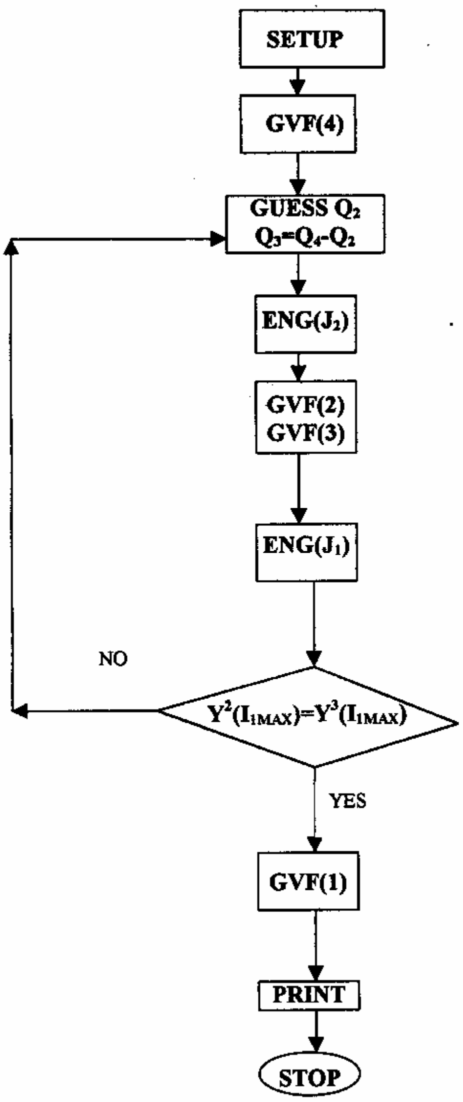


Fig. 5 Solution strategy

STEP 7: Compare the above two values of depth of flow at the downstream end of channel 1. If different, modify the guess for Q_2 in STEP 3 and repeat STEP 3 through STEP 6 till convergent values are obtained.

STEP 8: Apply the GVF solver to compute $y(I_1)$.

As evident from the above solution strategy, three different units, SETUP, GVF and ENG are important numerical steps and therefore, are explained below, in detail. The modification to the guess value of Q_2 is also presented.

3.2.3 SETUP

The input values are; length of each of the channel, (L_N), bed slope of each of the channel, (S_{bN}), bed roughness for each of the channel, (n_N), number of divisions in each of the channel, (D_N), discharge at the upstream end (Q_u) and the flow depth at the downstream end (y_d).

The discrete values of area, $A_m(I_N)$, top width, $T_m(I_N)$, and wetted perimeter, $P_m(I_N)$ with respect to height, $h_m(I_N)$ are given as input for known sections. For these sections, the values are interpolated for more number of heights. Then, for all the computational nodes, the values are calculated by interpolation procedure for a given depth of flow. However, in case of wide rectangular channels (as assumed in parametric studies), only the widths of individual channels are read as input values. The grid sizes and the maximum nodes are computed, for example, in case of channel *1*,

$$\Delta X_1 = L_1/D_1 \tag{8}$$

and

$$I_{1MAX} = D_1 + 1 \tag{9}$$

The length and slopes of channels *2* and *3* should be consistent as they share a common starting and end point i.e. the relationship

$$S_{b2}.L_2=S_{b3}.L_3 \qquad (10)$$

should be satisfied. Therefore, only three of the four variables are given as input and the fourth one is calculated.

3.2.4 GVF

This unit solves the ordinary differential equation for steady state gradually varied flow (Eq. 2) by Fourth Order Standard Runge-Kutta method. The equation is discretized and the differential equation is transformed to an algebraic equation. Given the discharge and the upstream flow depth, flow depths at all other nodes are computed. The equation is represented as a function of x and y. Thus for a node i,

$$f(x_i,y_i)= \text{right hand side of Eq.2} \qquad (11)$$

The calculational cycle is;

$$k_1=f(x_i,\ y_i) \qquad (12)$$

$$k_2=f(x_i-1/2.\Delta x,\ y_i+1/2.k_1.\Delta x) \qquad (13)$$

$$k_3=f(x_i-1/2.\Delta x,\ y_i+1/2.k_2.\Delta x) \qquad (14)$$

$$k_4=f(x_i-\Delta x,\ y_i+k_3.\Delta x) \qquad (15)$$

$$y_{i-1}=y_i+1/6(k_1+2.k_2+2.k_3+k_4)\Delta x \qquad (16)$$

For a given channel reach between nodes $I-1$ and I, depth of flow at the downstream, y_i is known. For this value of y_i, the corresponding A, T, P values at x_i are found out by interpolation from the stored discrete values. Thus, k_1 (eq. 12) can be found out. Then, $y_i+1/2k_1\Delta x$ is calculated. The values of A, T, P corresponding to the above value at

$x_i - 1/2\Delta x$ can similarly be found out and k_2 can be computed. Proceeding in a similar way the y_{i-1} can be found out as given in the above equations (Eq. 11 to 16).

This method is fourth order accurate. The function is known at the last node. The *y* values are evaluated at all other nodes by the above method. The solution progresses in the upstream direction. As we assume a subcritical flow in the channel the control point is at the downstream end.

3.2.5 ENG

The energy equation for the junction is solved by this unit. As given in Eq.6, the relationship is for six variables. Given the values of y_2, Q_2, z_2, y_1 and Q_1 the objective is to find y_1. The cross-sectional area, A_2, corresponding to the flow depth, y_2, is found out by interpolation of the stored discrete values. First y_1 is assumed. A_1 corresponding to this value of y_1 is interpolated. The functional value of the left hand side of eq.6 is found out.

$$G = y_1 + z_1 + \frac{Q_1.Q_1}{2.A_1.A_1.g} - (y_2 + z_2 + \frac{Q_2.Q_2}{2.A_2.A_2.g} + E_l)$$

If G is zero then assumption of y_1 is correct, else, the modification to the y_1 value is done by the following procedure. Let y_{11} gives G_1 and y_{12} gives G_2 as the functional values, respectively. Therefore, derivative of the function,

$$dG = (G_2 - G_1)/(y_{12} - y_{11}) \qquad (17)$$

The new guess for y_1 will be

$$y_1^n = y_{11} - G_1/dG \qquad (18)$$

The procedure is repeated till $G \approx 0$ is obtained. This is a highly efficient method and ascertains convergence very quickly. In case of wide rectangular channels, **G** is a function of y_1. Therefore, the solution can be obtained directly by using Newton-Raphson method for non-linear equations. The equation is given as;

$$y_1^n = y_1 - G(y_1)/G'(y_1) \qquad (19)$$

In the present mathematical model, E_1 is assumed to be zero. However, any suitable value of E_1 can be used if necessary. The initial guess value for y_1 is y_2.

3.2.6 GUESS Q_2

The value of Q_2 is guessed in the same manner as is done for y_1 in the solution of the energy equation. Considering STEP 6 of the solution procedure (see 3.22 above), an assumed value of Q_2 gives two values of the flow depth in the last node of channel *1* when calculated separately by applying the energy equation between channel 1 and channel 2 and channel 1 and channel 3. These two values should be same, otherwise, the value of Q_2 is modified.

Let energy equation between channel *1* and *2* gives the flow depth as fd_1 and the channel *1* and *3* gives fd_2, respectively. Error, *e*, can be computed as;

$$e = fd_2 - fd_1 \qquad (20)$$

If Q_2^1 gives e^1 and Q_2^2 gives e^2, then,

$$\Delta e = (e^2 - e^1)/(Q_2^2 - Q_2^1) \qquad (21)$$

The new guess for Q_2 will be,

$$Q_2^n = Q_2^1 - e^1/\Delta e \qquad (22)$$

The procedure is repeated till $e \approx 0$ is obtained.

3.3 LIMITATIONS OF THE MODEL

1. The flow is assumed to be one-dimensional which is not true at least for the channel junction. Also, due to the curvature of the channels there may be transverse flows.

Therefore, the model is only valid for small angles and low values of curvature of the branches.

2.The governing equations are for gradually varied flows. Therefore, the model is valid only for gradual transitions for width and bed elevations. The model is also not suitable for flows with sharp stream line curvatures such as in case of a bore or a hydraulic jump.

3.The model should not be used for spatially varied flows where the discharge changes with distance. Thus lateral inflows/outflows and porous medium flows are avoided.

4. The side wall effects are not considered.

6.Constant values of roughness coefficient.

3.4 APPLICATION TO FIELD SITUATIONS

Before applying the model to field situations, the following points may be considered.

(1) A channel is hardly straight and prismatic. Therefore, the channel should be divided into a number of reaches which are approximately prismatic and it should be treated as a series combination of channels.

(2) The area of cross section, the top width and the wetted perimeter should be calculated as a function of height for as much sections as possible.

(3) Channel bed roughness is a complicated function. Therefore, use of a constant value may result in erroneous results. The effective roughness for each section as a function of bed material, flow depth, bed slope and flow velocity should be used. Roughness values for the channel and the flood plain must be evaluated.

(4) The downstream end of the last channel should be far away from the junction such that the back water effects are not felt.

(5) Before applying to practical cases, the model should be calibrated against either field data or laboratory data taking scale effects into account.

4.0 RESULTS AND DISCUSSION

4.1 VALIDATION

The mathematical model presented in the previous section is used to study a single island type flow. A parametric study for three different cases of single island is performed for channels with wide rectangular section and for channels with irregular cross sections. However, before applying to these cases, the model is validated using previous results for a single reach channel. The performance of the present model is also compared with that of a previous model (FESWMS).

4.1.1 SINGLE REACH

The performance of the present model is validated with the experimental results for a single reach. Lansford and Mitchell (1949) conducted experiments in a flume and measured the water surface profile for a **M1** back water curve. The channel bed slope was 0.003 and roughens 0.019. The discharge was 0.226 m^3/s, the channel length was 91.4 m and the downstream flow depth was 0.319 m. The surface profile was computed for the above data using the present mathematical model. The comparison between the present model with the measured values is presented in Fig.6. The match is satisfactory and it builds confidence in the present mathematical model for further application.

4.1.2 COMPARISON WITH PREVIOUS MODEL

FESWMS is a commercially available software. It is capable of handling various types of free surface flow situations. It uses the Finite-Element method for the solution of the governing equations assuming shallow water theory. The input data, for a branched flow case, used for both the models are; $Q_1 = Q_4 = 1000\ m^3/s$, $L_1 = L_4 = 2000m$, $L_2 = L_3 = 40000m$, $n_1 = n_2 = n_4 = 0.03$, $n_3 = 0.02$, $B_1 = B_2 = B_4$

$=500m$, $B_3 = 100m$, $S_{b1} = S_{b2} = S_{b3} = S_{b4} = 0.0001$. Considering the division of flow, Q_3 is predicted as 229.508 m^3/s and 230.0 m^3/s, respectively, by the present model and FESWMS. The comparison of the water surface profile for Channel-2 is shown in Fig.7. The comparison is satisfactory. However, considering the CPU time, ease in giving the input data and accuracy of results obtained for the simple case study, the present model may be preferred for application to the proposed study of a single island type flow.

4.2 PARAMETRIC STUDY

The mathematical model is used to study the flows in (i) a diversion channel, (ii) a cutoff channel, and, (iii) a river island. A brief introduction followed by the input parameters and the results are presented. The input values are fictitious but closely approximate the field values. The channels are assumed to be wide rectangular. This will help in deriving important generalized results. All the results are presented in a non-dimensional manner.

Non-dimensional discharge = $Q^* = Q_3/Q_1$, Non-dimensional width = $B^* = B_3/B_2$, Non-dimensional bed slope = $S^* = S_{b3}/S_{b2}$, Non-dimensional roughness = $n^* = n_3/n_2$ and Non-dimensional flow depth = $Y^* = y_d/Y_{dn}$ (y_{dn} is the normal flow depth at the downstream end) are used in this study.

As mentioned earlier, in all the studies, the bed is assumed rigid, i.e. the migration of the confluence point, the erosion of banks and aggradation/degradation of the bed are not considered.

4.2.1 DIVERSION

A diversion channel is generally man-made and serves the purpose of protecting the area from flood. It carries the extra flow safely round the area to be protected (Fig.8). The length will be generally more than the main channel and therefore, its bed slope will be less.

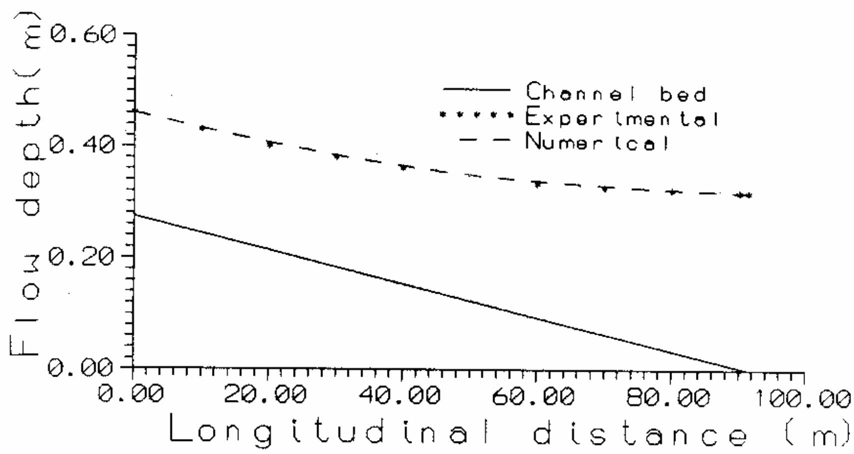


Fig.6 Validation for a single reach channel

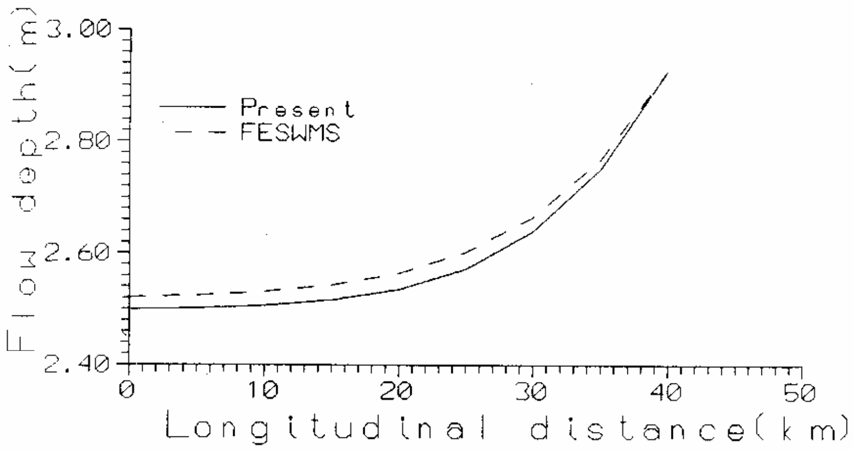


Fig.7 Comparison with FESWMS

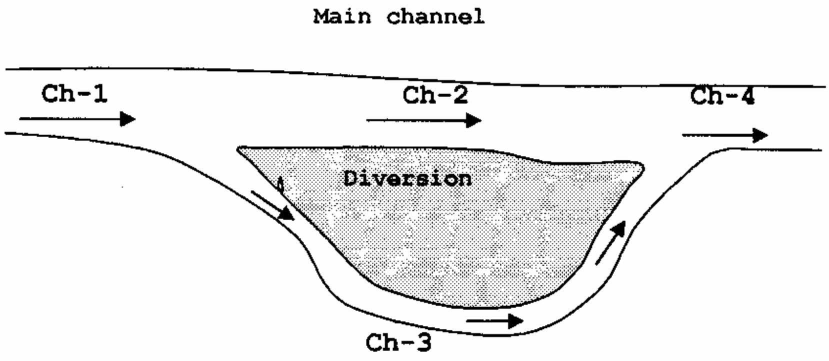


Fig. 8 Diversion channel

The cross-sectional area may be assumed to be regular. The bed roughness may be either higher, same or lower than that of the main channel. The width will be less than that of the main channel.

INPUT VALUES: The parameters of channels 1,2 and 4 are kept constant (Ref. Fig.8). The parameters of channel 3 are varied and the effect of these parameters on the flow division is studied. The input values are;

$L_1 = L_4 = 1$ *km*, $L_2 = 60$ *km*, $L_3 = 90$, 120 or 150 *km*, $S_{b1} = S_{b2} = S_{b4} = 0.0001$, $B_1 = B_2 = B_4 = 500$ *m*, $B_3 = 50$ to 500 *m*, $Q_1 = Q_4 = 1000$ *m3/s*, $I_{1MAX} = I_{4MAX} = 10$, $I_{2MAX} = 600$, $I_{3MAX} = 900$, 1200 and 1500, $n_1 = n_2 = n_4 = 0.030$, $n_3 = 0.015$, 0.030, 0.045. S_{b3} is calculated by Eq.10 and y_d is the normal flow depth corresponding to Q_4.

RESULTS: Effect of non-dimensional width on non-dimensional discharge for different values of non-dimensional roughness coefficient is shown in Fig.9. As expected, more the width in Channel-3, more is its share of flow. Similarly as roughens coefficients increase in channel-3, the discharge through it decreases. For all the computations presented in this figure, a constant non-dimensional bed slope = 0.666. It corresponds to the diversion channel (Channel-3) is 1.5 times longer than the main channel (channel-2.). In Fig.10, the variation of the non-dimensional discharge with the non-dimensional width is shown for different values of non-dimensional bed slope. A constant non-dimensional roughness coefficient = 1.0 is used in the same figure. This figure indicates if the bed slope in channel-3 is higher, flow through it is more. In both Figs. 9 and 10, the non-dimensional width is varied from 0.1 to 1.0. The maximum value of flow diversion is approximately 50%. Thus, for a wide range of given parameters (geometry and roughness), the flow through the diversion can be predicted.

Using the above results, a regression analysis is performed to derive a simple relationship between the parameters for a diversion channel. The derived relationship is;

$$Q^{*}=0.4\,(B^{*}/n^{*})^{1/2}\,(S^{*})^{1/10} \qquad (23)$$

The above equation is with $R^2 = 0.986$. The computed and the fitted values are presented in Fig.11.

SENSITIVITY ANALYSIS:

All the results obtained above are with definite values of individual parameters. But, the relationship (Eq.23) has only the non-dimensional parameters. The predictability of the above equation (Eq.23) is verified through a sensitivity analysis in order to find the effect of the parameters.

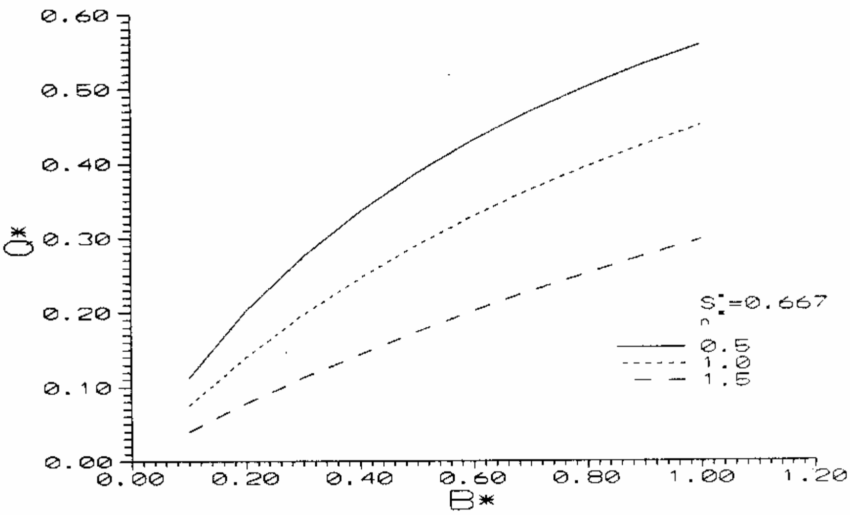


Fig. 9 Effect of width on flow distribution for different roughnesses(Diversion)

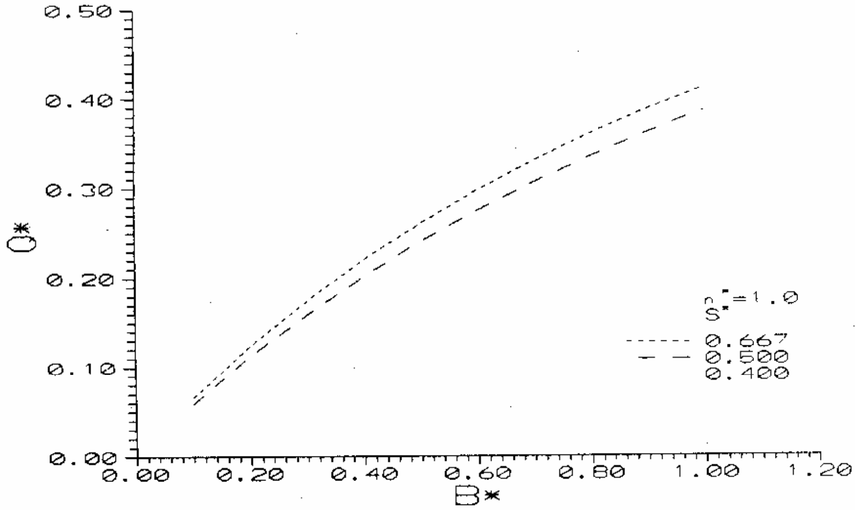


Fig.10 Effect of width on flow distribution for different slopes(Diversion)

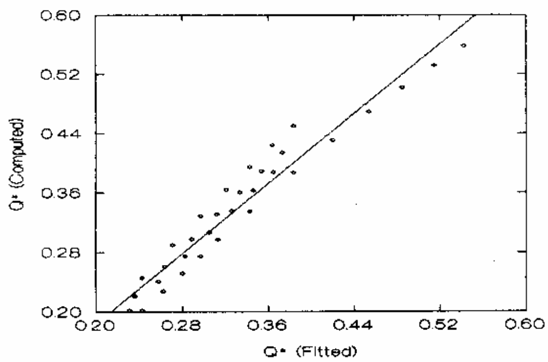


Fig.11 Comparison of computed and fitted discharges (Diversion)

Fig.12 shows the effect of inflow discharge, Q_1 on the non-dimensional discharge, Q^*. The figure indicates a very small variation (0.4 %) on the Q^* value for a wide range of Q_1 values. Therefore, Eq.23 is valid for any value of Q_1. In all the computations mentioned in the above paragraph, the downstream flow depth used was computed assuming a normal flow condition. The actual stage-discharge relationship should be used. Therefore, the downstream flow depth is varied to see the effect on flow division (Fig.13). For the given conditions ($n^* = 1.0$, $S^* = 0.666$, $B^* = 1.0$ and $L_4 = 2\ km$), y_d has no effect on Q^* till $Y^* = 2.7$. Therefore, Eq.23 is valid for $y_d = 2.7y_{nd}$. for this set of data. For higher values of y_d, there will be a backwater effect, and Q^* should be computed using the mathematical model. Effect of L_2 keeping S^* unchanged is shown in Fig.14. The variation is negligible after a critical value is attained. Therefore, a very low value of

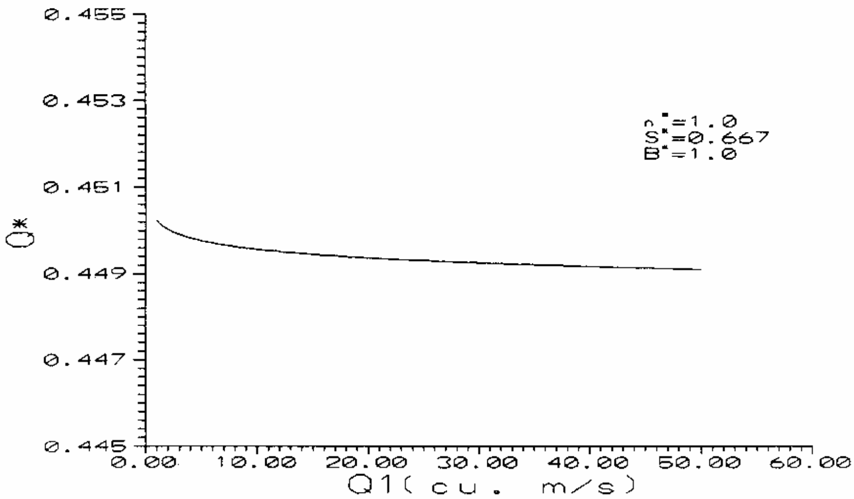


Fig. 12 Effect of inflow, Q_1, on flow distribution (Diversion)

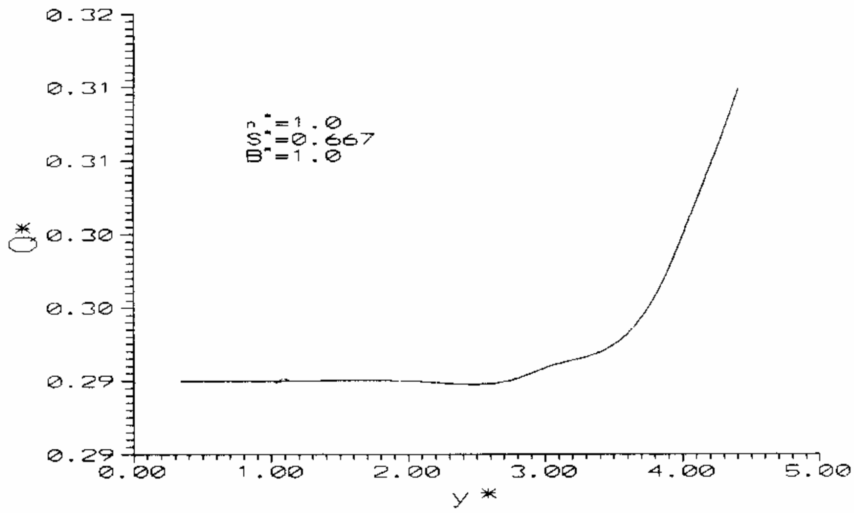


Fig. 13 Effect of downstream depth on flow distribution (Diversion)

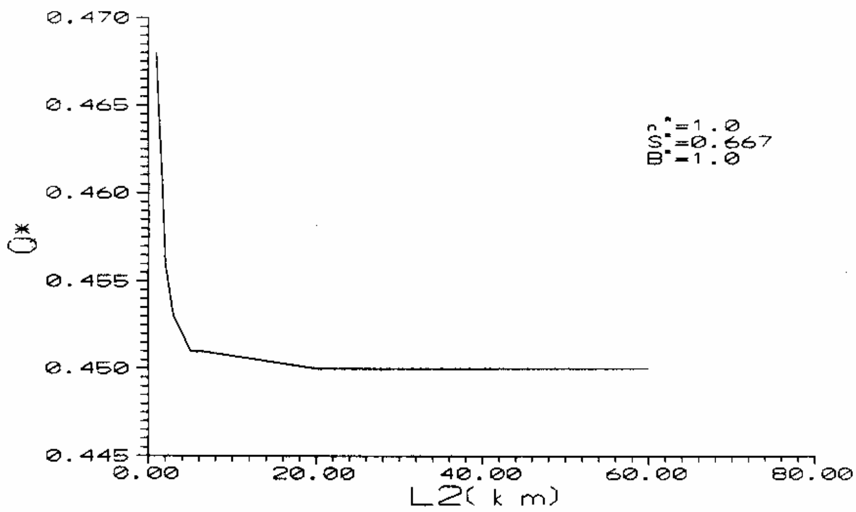


Fig. 14 Effect of length, L_2, on flow distribution (Diversion)

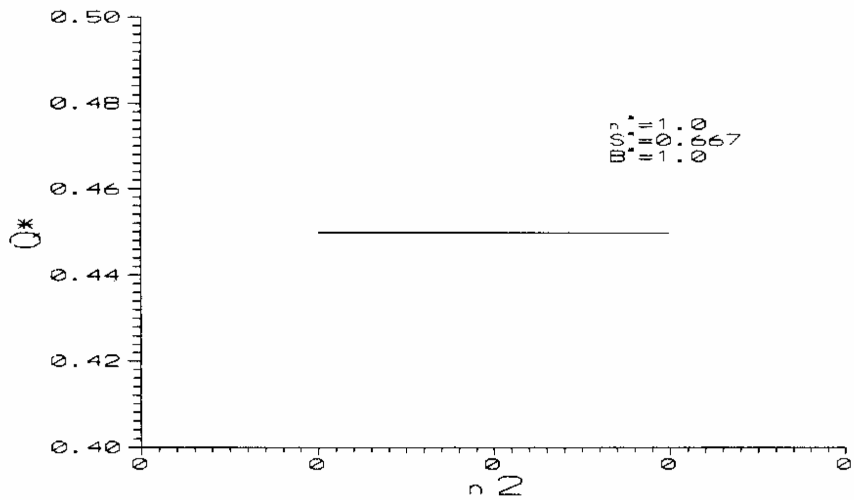


Fig. 15 Effect of roughness, n_2, on flow distribution (Diversion)

L_2 should not be used. In other words, Eq.23 will not be applicable if L_2 is very small (in this case less than 5 km). Fig.15 shows there is no effect of the roughness n_2 on the discharge division. Therefore, Eq.23 is valid for any value of n_2.

4.2.2 CUTOFF

Artificial cutoffs are built across the highly meandered stream in order to increase its hydraulic efficiency (Fig.16). The flow along the two channels must be balanced to give the same head loss over the section. When river improvement works are executed the values of roughness coefficient change. By decreasing the length of the stream, the bed slope of the cutoff increases resulting in a higher velocity of flow. Due to increased bed slope and velocity, peak floods reach downstream more rapidly. The increase in velocity results in a lowering of flood heights. Consequently, the risk of flood upstream of a cutoff is decreased. However, the portions of the flood plains downstream of the cutoff may be more liable to flooding. The change in the direction of the current as it comes out of the cutoff as compared with its direction in the natural condition may affect the bed and banks.

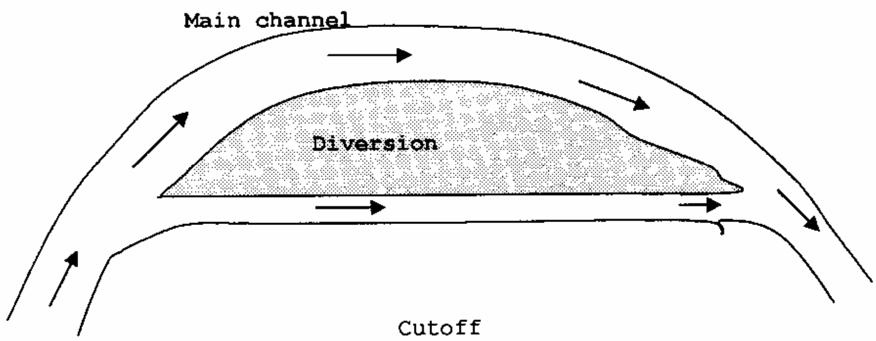


Fig. 16 Cutoff channel

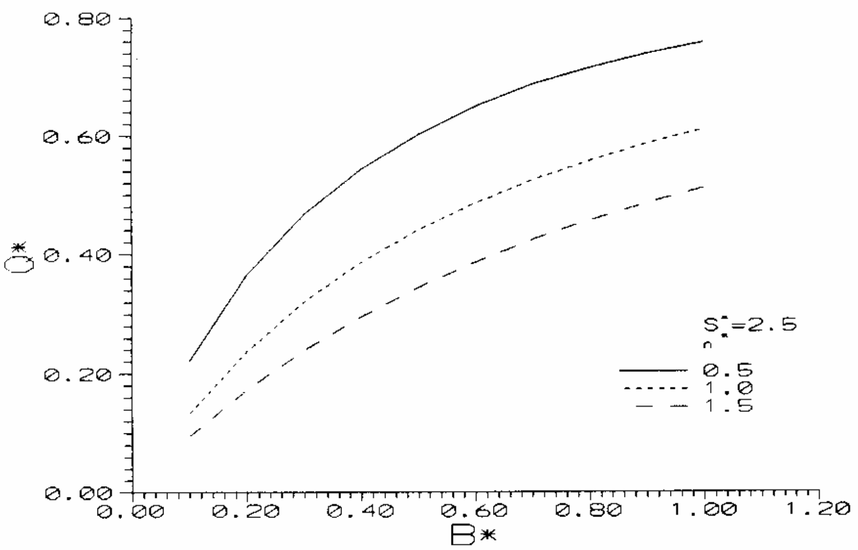


Fig. 17 Effect of width on flow distribution for different roughnesses (Cutoff)

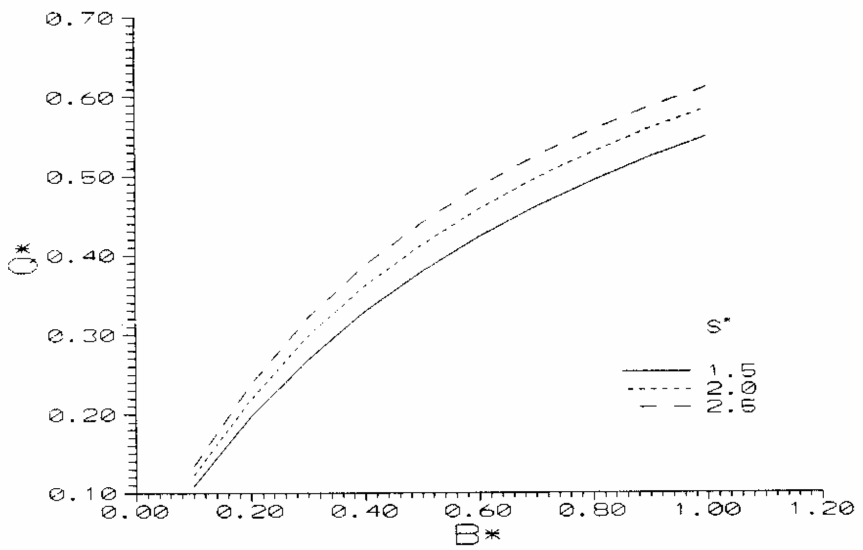


Fig. 18 Effect of width on flow distribution for different slopes(Cutoff)

However, in the present work the flow is analysed assuming a rigid bed.

INPUT VALUES: The parameters of channels *1,2* and *4* are kept constant. The parameters of channel *3* are varied and the effect of these parameters on the flow division is studied. The input values are; $L_1 = L_4 = 1\ km$, $L_2 = 60\ km$, $L_3 = 40$, 30 or 24 km, $S_{b1} = S_{b2} = S_{b4} = 0.0001$, $B_1 = B_2 = B_4 = 500\ m$, $B_3 =$ 50 to 500 m, $Q_1 = Q_4 = 1000\ m3/s$, $I_{1MAX} = I_{4MAX} = 10$, $I_{2MAX} =$ 600, $I_{3MAX} = 400$, 300 and 240, $n_1 = n_2 = n_4 = 0.030$, $n_3 =$ 0.015, 0.030, 0.045. As in the case of the diversion - channel, S_{b3} is calculated by Eq.10 and y_d is the normal flow depth corresponding to Q_4.

RESULTS: Analogous to Figs. 9 and 10 presented earlier in case of diversion channel, Figs. 17 and 18 show the effect of non-dimensional roughness coefficient and non-dimensional bed slope, respectively, on the flow division for various widths. The relationship between the parameters for a cutoff channel is;

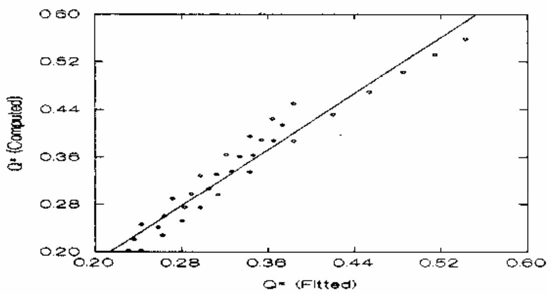


Fig. 19 Comparison of computed and fitted discharges (Cutoff)

$$Q^*=0.5(B^*/n^*)^{1/2}(S^*)^{1/6} \quad (24)$$

The above equation (Eq.24) is with $R^2 = 0.992$. The computed vs. the fitted values are shown in Fig.19.

4.2.3 RIVER ISLAND

River islands are built naturally. It may be with a braided system giving rise to multi-island flow. A river island is always associated with problems of sediment transport. Bank erosion, channel bed aggradation /degradation are common phenomena in a river island. However, in this work, only a simple island flow with a rigid bed is assumed in the flow analysis (Fig.20). The length of the branch channel (Channel-3) can be either more than, equal to or less than the length of the main channel (channel-2). Similarly, the width, the roughness coefficient and bed slope values can vary. Previous results (Figs. 9, 10, 17 and 18) can be used for a river island. However, The effect of non-dimensional width on the non-dimensional discharge for various non-dimensional roughness coefficients using a constant non-dimensional bed slope = 1.0 is shown in Fig. 21. Combining all the results (Figs. 9, 10,18, 19 and 21) the following relationship is derived for the river island type flow.

$$Q^*=0.455(B^*/n^*)^{1/2}(S^*)^{1/3} \quad (25)$$

The above equation is with $R^2 = 0.996$ and the computed vs. the fitted results are shown in Fig.22.

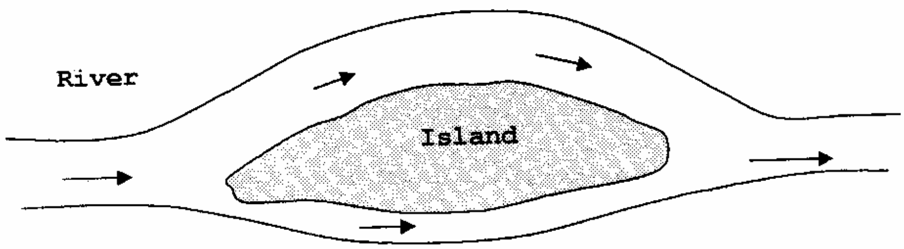


Fig. 20 A river island

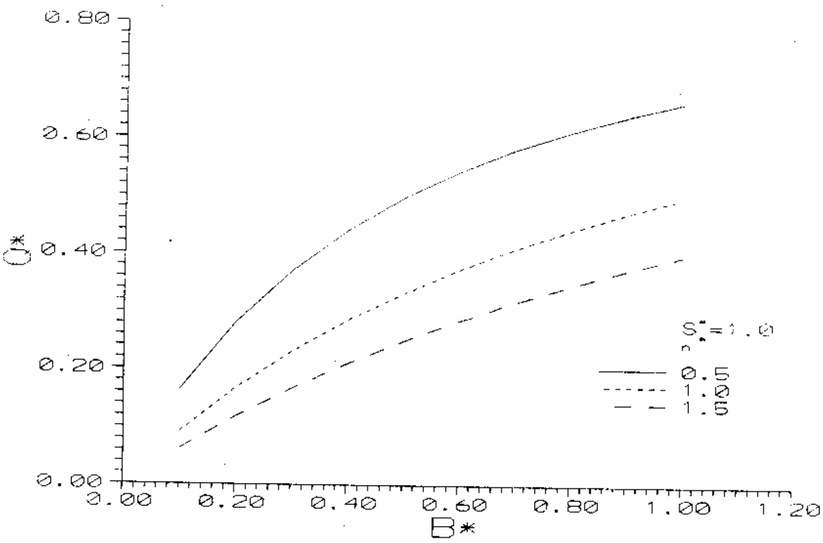


Fig. 21 Effect of width on flow distribution for different roughnesses (River island)

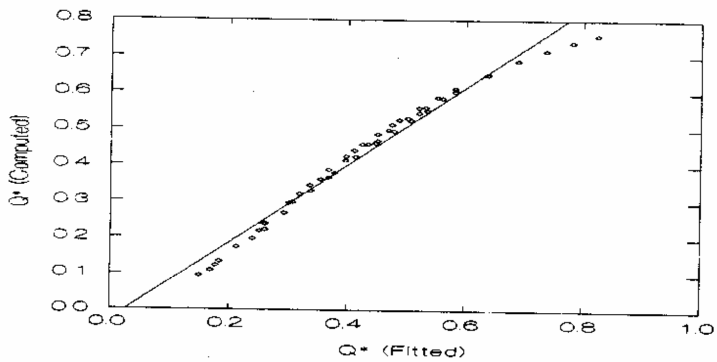


Fig. 22 Computed vs. fitted discharges (River island)

4.3 ISLAND WITH IRREGULAR CROSS-SECTION

In this subsection, a simple island assuming highly irregular cross-sections is considered for the analysis. The objective is to show the versatility of the present mathematical model. In this study, first a straight reach is analysed without any branching. Then the same reach with a branched channel is considered. Comparison of the same study case assuming a rectangular cross-section is also shown. The plan of the main channel with branch is shown in Fig.23. The cross-sections are known at three different places. Variation of the area of cross-section (A), top width (T) and wetted perimeter (P) with respect toheights(h) at three different places are presented in TABLE-1.These values are read into the present mathematical model. The model calculates the corresponding values at all the computational nodes by linear interpolation. Other input values are; Q = 3000 m^3/s, L = 40 km, n = 0.03, S_b =

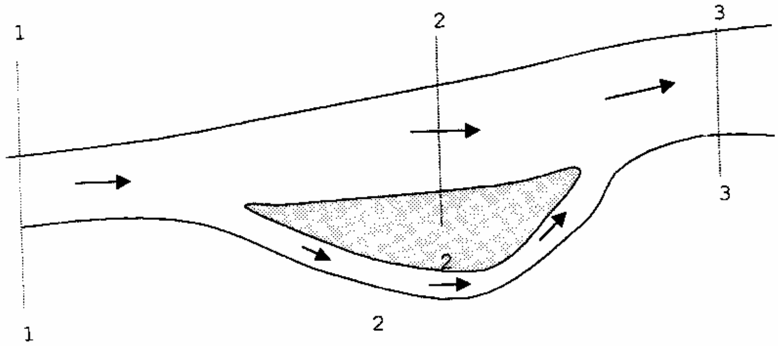


Fig. 23 A diversion with irregular cross-sections

0.0001 and y_d = 8.0 *m*. The branch channel is started at a distance of 4 *km* from the beginning and it merges with the main channel again at a distance of 36 *km*. The length of the branch channel is assumed to be 44 *km*. The branch channel is assumed to be of rectangular cross-section with a width of 200 *m* through out. The roughness coefficient in the branch channel is 0.02. The surface profile in the main channel with and without the branch channel is shown in Fig.24. The discharge is reduced to 1799.6 m^3/s in the presence of the branch channel.

In order to get an idea of the performance of the derived relations (Eqs. 23, 24 and 25) for cases with irregular cross-sections the following procedure is adopted.

The main channel is assumed to be rectangular and the width is determined by

TABLE-1 Different cross-sections

H	A	T	P
	Cross-section 1-1		
2.0	300	140	250
4.0	700	291	400
8.0	2200	580	690
13.0	5050	650	720
	Cross-section 2-2		
1	150	100	180
3	500	200	350
5	930	425	460
7	2000	600	700
10	3200	650	725
15	6000	680	740
	Cross-section 3-3		
2.8	680	316	330
4.8	1400	508	520
6.8	2560	700	742
12.8	6932	740	766

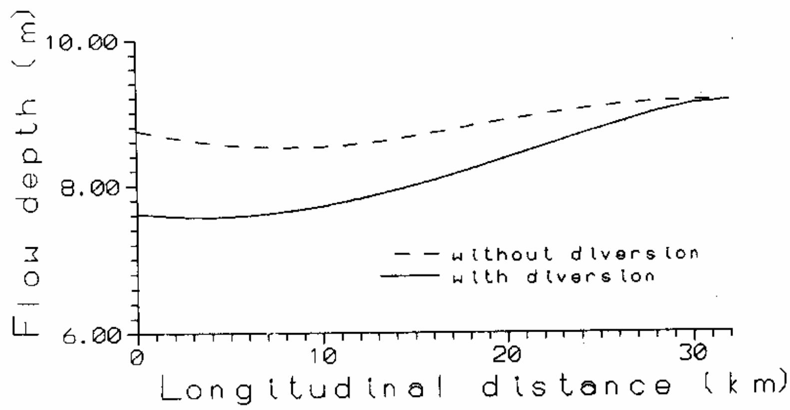


Fig. 24 Surface profile for channel-2

$$B = \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} A_{i,j} / h_{i,j} \qquad (26)$$

where, B is the average width of the channel, m is the number of stored values at any cross section and n is the number of cross sections. With the above B value and all other input values, Q_2 is found to be 1759.7 m^3/s using Eq.23. This shows that the derived equations can also be used with confidence.

5.0 CONCLUSION

In this work, a single island type of flow with a rigid bed has been studied for a steady flow condition. The present mathematical model is accurate, efficient, and robust. The main conclusions of the present study are;

1. The flow division due to branching depends on the inflow characteristics, the channel geometry and bed conditions.

2. A parametric study shows the effect of branch width on flow division due to different values of bed roughness and bed slopes. Simplified relationship in equation form has also been derived.

3. A study of an island with irregular geometry suggests, the simplified equations can be used for irregular geometry after averaging the irregularity.

Recommendations for future study

Although the present mathematical model is working satisfactorily for simple study case considered here, following extension to the work in future is recommended in order to make the model a potent tool for analyses of branched flows.

(1) The model should be extended to two-dimensional cases. (2) Time variation of different flow parameters should be computed by unsteady flow modelling. (3) Modelling of the bank erosion and channel bed aggradation/degradation should be performed. (4) Depth-averaged turbulence modelling (K-ε model) should be included to account the flow patterns near the junction.

REFERENCES

Biron, P., Best, J. L., and Roy, A. G. (1996)"Effects of Bed Discordance on Flow Dynamics at Open Channel Confluences", *Jl. of Hyd. engrg.*, ASCE, 122(12), 676.

Bowers, C.E.(1950) "Studies of Open Channel Junctions", Tech. Paper No.6, Ser.B, (Part v of Hyd. Model Studies) Mitton, Florida.

Chaudhry, M.H. (1993) - *Open Channel Flow*, Prentice Hall, Englewood Cliffs, N.J.

Chaudhry, M. H. and Schulte, A. (1986) "Computation of Steady-state Gradually Varied Flows in Parallel Channels", *Canadian Jl. of Civ. Engrg.*, 13(1),39-45.

Choi and Molinas (1993) "Simultaneous Solution Algorithm for Network",

Christodoulou, G. C. (1993) "Incipient Hydraulic Jump at Channel Junctions", *Jl. of Hyd. Engrg.*, ASCE, 119(3),

Cunge, J.A., Holly,F.M. and Verwery, A. (1980) *Practical Aspects of Computational River Hydraulics*, Pitman, London.

Grace, J.L. and Priest, M.S. (1958) "Division of Flow in Open Channel Junctions", Bulletin No.31, EES,. Alabana Polytechnic Institute.

Gurram, S. K., Karki, K. S. and Hager, W. H. (1997) "Subcritical Junction Flow", *Jl. of Hyd. Engrg*, ASCE, 123(5), 447.

Hager, W.H. (1983) "Approximate Treatment of Flow in Branches and Bends", Proc. Inst. Mech. Engr. Part C, 198(4), 63-69.

Hager, W.H. (1989) "Subcritical Flow in Channel Junctions", *Jl. Hyd. Engrg.*, ASCE, 115(5), 559-616.

Krishnappa, G. and Seetharamiah, L. (1963) "A New Method of Predicting the Flow in a 90^0 Branch Channel" 775-778.

Lansford and Mitchel (1949) "An Investigation of the Backwater Profile for Steady Flow in Prismatic Channels"

Law, S.W. and Reynolds, A.J. (1966) "Dividing Flow in Open Channel" *Jl. Hyd. Div.*, ASCE 92 (HY2), 207-231.

Li, Z., Zhan, L. and Wang, H.(1983)"Difference Methods of Flow in Branch Channel", *Jl. Hyd. Engrg.*, ASCE, 109(3),424.

Liggett, J.E. (1993) *Fl d Mechanics*, Mc GRaw Hill.

Milne - Thomson, M. (1949) "On the Flow Characteristics of a Channel with a Distributary", Tech. Reports Osaka Univ. Japan, 6(198).

Naidu, B. J. Bhallamudi , S. M. and Narasimhan, S. (1997) "GVF Computation in Tree-type Channel Networks",*Jl. Hyd. Engrg*, ASCE, 123(8),700.

Neary, V. S. and Odgaard, J. (1993) "Three-Dimensional Flow structure at Open Channel Diversion", *Jl. Hyd. Engrg.*, ASCE, 119(11), 1223.

Nguyen Q. K. and Kawano, H.(1995) "Simultaneous Solution for Flood Routing in Channel Networks", Jl. Hyd. Engrg., ASCE, 121(10),744.

Pattabhiraniah, K.R. and Rajaratnam, N (1960) "A New Method of Predicting Flow in a Branch Channel", *Jl. Central Board Irrig. and Power*, 17(1), 48-51.

Petersen, M.S. (1986) *River Engineering*, Prentice Hall, Englewood Cliffs, N.J.

Rajaratnam, N. (1967) "The Improvement of the Inlet of the Power Canal", Trans. 7th Gen. Meeting of IAHR, Vol1, C:20-1-20-16.

Ramamurthy, A.S. and Satish, M.G. (1988) "Division of Flow in Short Open Channel Branchges", *Jl. of Hyd. Engrg.*, ASCE, 114(4), 428-438.

Ramamurthy, A.S. Tran, D. M. and Carballada, L. B. (1990) "Dividing Flow in Open Channels", *Jl. of Hyd. Engrg.*, ASCE, 116(3), 449-455.

Satish, M.G., Ramamurthy, A. S., and Narasiah, K. S. (1987) "Pressure Recovery in Dividing Open Channels", Jl. of Hyd. Engrg., ASCE, 115(7),995.

Schulte, A. M. and Chaudhry, M. H. (1987) "Gradually varied Flows in Open Channel Networks", Jl. of Hyd. Res., IAHR,25(3),357-371.

Stoker,J J.(1957) Water Waves, Interscience.

Subramanya, K. (1985)Flow in Open Channels, Tata McGrawhill.

Taylor, E (1944) "Flow Characteristics at Rectangular Open Channel Junctions", Trans., ASCE, 109, 893-902.

Yen, B.C. (1979) "Unsteady Flow Mathematical Modeling Techniques", Chap.13 in *Modeling of Rivers*, Ed. Shen, H.W., John Wiley and Sons, New York.

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