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**MATHEMATICAL MODELING OF BRANCHED FLOWS IN
ALLUVIAL CHANNELS**



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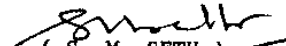
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PREFACE

The computation of branched flows in alluvial channels, quite common in nature is of paramount importance in various flood management programmes. Man-made branching may be done to enhance the performance of a stream. Most of the models to study branched flows are for rigid bed condition whereas in reality the branched flow occurs in alluvial conditions. Problems like erosion of river islands and formation of deltas can never be attempted by such rigid bed models. Analytical models are for highly simplified cases and physical models are expensive and time consuming. Mathematical models have become popular due to advent of fast computers. However, mathematical modeling of branched flows in alluvial conditions is not easy and straightforward. Unlike the water flow dynamics, where equations are well established, sediment flow dynamics is not properly understood. Almost all sediment discharge relations are empirical by nature and their success depends on local conditions. Moreover, the bed roughness characteristics change a lot due to transport of sediments. The interplay between water flow, sediment-discharge and bed roughness is still a subject of research. The importance of analysis of alluvial streams is increased due to transport of pollutants by sediments.

In this report, a detailed review is prepared for the mathematical modeling of flows in alluvial condition. Various computer codes for the above purpose are presented. A model for simple-island case is also proposed.

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ABSTRACT

Branched flows in open channels with alluvial conditions occur in natural and man-made systems. Analysis of such flows by mathematical modeling is important considering its applications in water resources and environmental engineering. In this report, a detailed review of existing models for flows in alluvial streams is presented. Models may be categorized based on (i) equations used, (ii) numerical method applied, and, (iii) consideration of physical processes. The governing equations for water flow and sediment flow are presented. The equations to be satisfied at junctions are also presented. Solution strategies and their variations are discussed. Various numerical methods for the solution of the governing equations are mentioned. Implementation of the boundary conditions is also described. A large number of existing computer codes to study flows in alluvial streams is presented. Performance of various models is problem dependent and no model is suitable for all types of problems. This is due to the poor understanding of the relation between flow and sediment transport. Thus, there is a scope to find the exact mechanism of sediment transport, which can be a potent area for future research. In addition, the roughness characteristics of the channel bed should also be estimated in the presence of sediment transport. Finally, a numerical scheme based on one-dimensional equations, a quasi-steady assumption and an uncoupled approach using explicit finite-difference method, is proposed. Although the model is expected to perform with greater efficiency and robustness, it is yet to be tested against measured data and needs further study.

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1.0 INTRODUCTION

Branched flows in alluvial rivers/channels are very common in natural and man-made systems and occur when the flow is divided into more than one separate channel. Tributaries flow into and distributaries flow away from the main river. Deltas are formed as streams deposit bed load and coarser suspended sediments where carrying capacity of streams is reduced. The stream gets braided if the bank material is easily eroded, if the sediment load is composed in large part of sands and gravel moving as bed load, and if the dunes that are formed are large (Petersen 1986). This gives rise to a very complex flow network. Branched flow may occur due to natural islands in a river. The irrigation system is always associated with branching due to the networking consisting of main canals, branch canals, minors and distributaries. Diversion channels are often constructed for flood control purposes. Prior to a dam construction, provision of a diversion tunnel is a common practice. Cut-off channels are either man-made or natural and are useful to protect the banks of a highly meandering river.

Branched flows can be either looped, where the branches join again at a downstream point, or non-looped where the branches do not meet again. Looped flows can be (i) single looped, and, (ii) multi-looped. The single looped flow is known as *Simple Island*, the multi-looped channel as *Multiple Island* and the non-looped flow as *Tree type flows*. Channel junctions can be divided into two categories, viz. point type junction and storage type junction (Yen 1979). For junctions with insignificant storage capacity, the junction can be considered as a point type junction. It is represented by a single confluence point without storage. The net discharge into the junction is, therefore, zero at all times. The reservoir type of junction is assumed to behave like a reservoir with a horizontal water surface. It is capable of adsorbing and dissipating all the kinetic energy of the flow. The net discharge into the junction is equal to the rate of change of storage in the junction. A typical branched flow in open channel is shown in Fig. 1.

The physical processes involved in a branched open channel flow in alluvial condition are highly complex. The flow is unsteady, three-dimensional and turbulent. Transverse and circulating flows are very prominent near the junction. If the branch channel is highly curved, the flow away from the junction may also have transverse components. Presence of a bore adds to the complexity as it poses a discontinuity in the water surface. Inclusion of either lateral inflows/outflows or a porous media makes the flow spatially varied. In an alluvial condition, the channel bed is mobile and dynamics of sediment plays a major role. It includes the processes of bank erosion and aggradation/degradation of channel bed. Transport of pollution may also take place. The time scale of flow variation is significantly different from that of bed level variation. The flow patterns are governed by the approaching flow, geometry of branches, bed conditions and characteristics of sediments. Flow, sediment transport and channel roughness/ channel geometry in a fluvial stream are interdependent. The flow affects the sediment transport, which controls the hydraulic roughness and channel geometry through bed-wave formation and sediment deposition/erosion. The hydraulic roughness and channel geometry, in turn, affects the flow. For example, it is possible to have multiple flow velocities and sediment transport rates for the same flow depth or discharge in a given channel, depending on the bed form. Thus, to make depth-discharge predictions for alluvial flows requires knowledge of the relationships among flow parameters, fluid/sediment properties, and the hydraulic roughness affected by sediment transport.

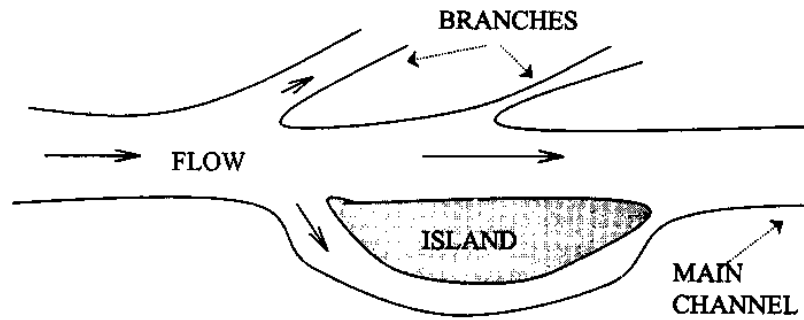


Fig. 1 Branched Flow

Here lies the critical difference between an alluvial flow and a rigid channel flow and the source of the difficulties dealing with alluvial streams.

In an alluvial environment, a channel has four degrees of freedom, i.e. flow depth, bed width, longitudinal slope and layout of the channel, all of which may undergo changes with space and time. Accurate analysis of flow variables in a branched flow is essential in many engineering applications. Estimation of the free surface elevation helps in the determination of the effect of hydraulic structures on upstream and downstream channels, estimation of flood zone and determination of the safe and optimum operation of control structures. The design of diversion channels and cut-offs greatly depend on the quantity of flow to be passed through them. Design of various structures to prevent bank erosion, scouring and deposition of channel bed also depends on such analyses. Hydraulic and environmental engineers have a great need to evaluate the transport, deposition, and re-suspension of sediment for the planning and operation of river and canal system. This requirement is made greater by environmental assessment and clean up activities, since fine sediments act as a carrier of many toxic chemicals, heavy metals, and radio-nuclides.

Analysis of a branched flow in open channels with alluvial bed conditions can be performed by physical modelling of the problem under consideration. A thorough knowledge of similitude analysis, flow visualisation techniques and flow measurements is necessary for a successful completion of the experiment. Laboratory as well as field models may be employed for the purpose. In physical modelling, very close implementation of the boundary conditions and flow geometry is possible. The results are reliable and greater understanding of the problem is achieved. However, the methods are costly and time consuming. Besides, scale effects may also be present. Though mathematical modelling is abundantly used, it can never replace physical modelling. A comparison between physical, analytical and numerical models is presented by Anderson et al. (1984).

Previous studies with respect to the branched flows show that most of the studies are for rigid beds. A detailed literature review on rigid bed branched flow modelling has been presented by Palaniappan and Mohapatra (1997). Also, there are many studies concerning experimental or analytical solutions for highly simplified cases. The temporal variation of the channel bed is also missing. Although, experimental and analytical studies contribute significantly to our understanding of open channel flows in alluvial conditions, mathematical modelling of

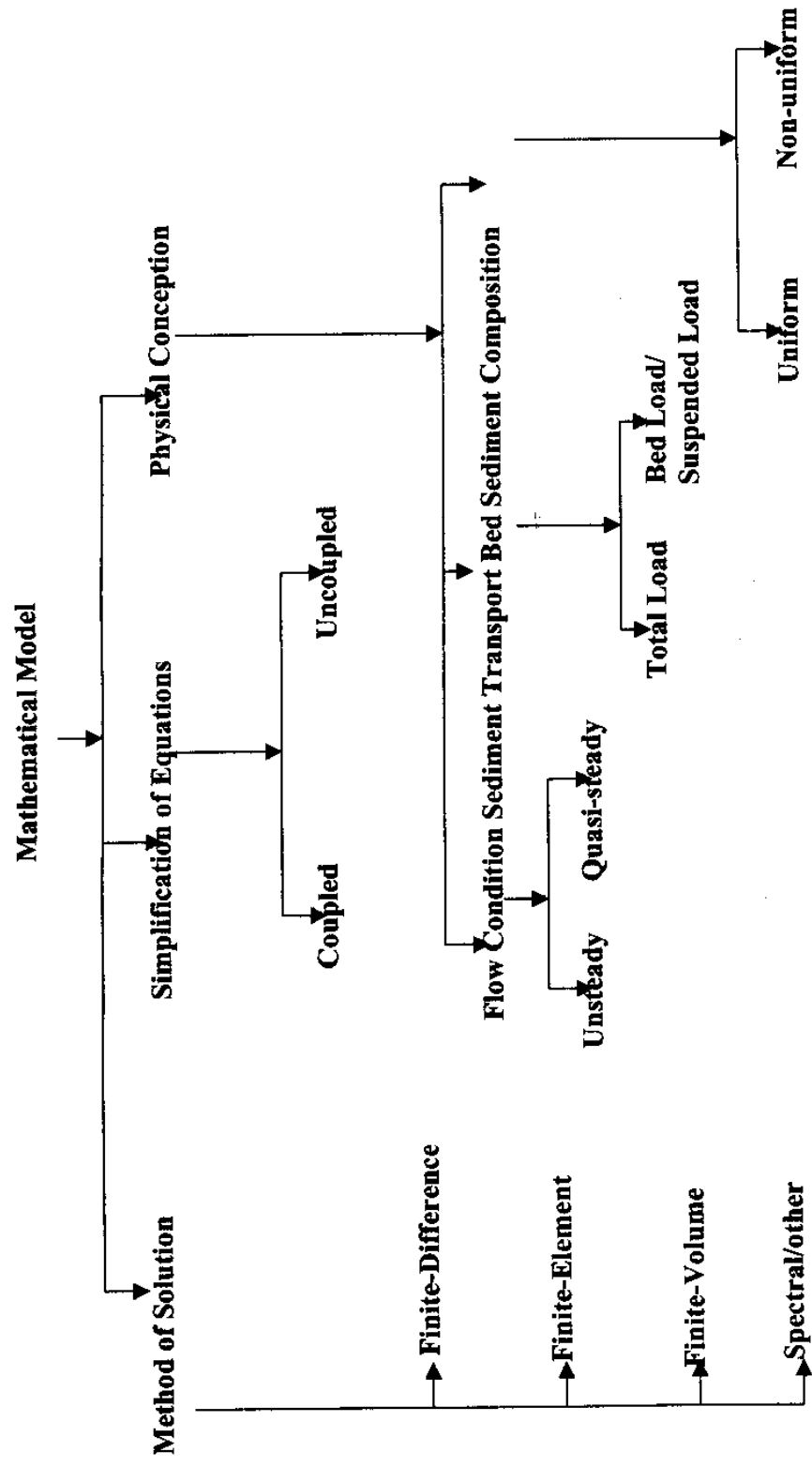


Fig. 2 Classification of Mathematical Models

Movable bed hydraulics has become an active area of research in computational hydraulics. Mathematical models for alluvial channels essentially solve the governing equations describing conservation of mass and momentum. The sediment flow is represented by the sediment continuity equation and a sediment transport equation which relates the sediment discharge to the flow parameters. This sediment transport equation is generally an empirical equation based on laboratory and/or field observations. The above governing equations for water and sediment constitute a set of non-linear hyperbolic partial differential equations for which analytical solutions are possible for simple cases only. Therefore, these equations are solved using numerical schemes. Either Finite-Difference or Finite-Element methods can be used for this purpose. Most of the standard models can be classified into either uncoupled models, wherein the water flow equations and sediment continuity equations are solved separately during a given time step or coupled models wherein all the equations are solved simultaneously. They can be classified as either unsteady models or quasi-steady models. The unsteady models solve the complete equations and quasi-steady flow models assume that water flow is steady during the computation of bed level variation. Models also differ with respect to the physical conception. Some models use uniform size sediment while others use non-uniform size sediment and attempt to simulate complicated processes like bed armouring. For the modes of transport of sediment, some use total load concept while others make a distinction between different mode of transport. A detailed classification of models is presented in Fig. 2. Numerous methods and formulas are available to predict stage-discharge relationships, and to calculate sediment transport rates in rivers. They are an integral part of numerical sediment transport models.

The main objectives of this report are;

(i) to review previous works pertaining to mathematical modeling of flow in open channels with alluvial conditions, and, (ii) to propose a model to study the simple island branch flow problem.

In this section, the occurrence of branched flows, their main characteristics in alluvial conditions and importance in engineering applications have been presented. Equations governing the flow in alluvial conditions and numerical methods to solve them are presented in sections 2 and 3, respectively. In section 4, various computer codes suitable for alluvial conditions are described in brief. In the last section, important conclusions of the present study and recommendations for future research are presented. A proposed model for the case of a simple island flow is presented in the Appendix.

2.0 GOVERNING EQUATIONS

To simulate the unsteady flow in a branched open channel with alluvial conditions, the governing partial differential equations for the flow of water and sediment are numerically solved. In addition, the energy equation and the characteristic equations are also to be solved at junctions. Although extensive research has been carried out to understand the exact relationship between water flow and sediment movement (Shen 1970, Garde and Rangaraju 1985), the present knowledge in this area can only be considered semi-empirical. In addition, complete solution of three-dimensional equations of motion for water and sediment is very complicated. Generally, following two major assumptions are made in the analysis.

- (1) Velocity of sediment particles is not important.
- (2) The sediment discharge at any point is uniquely related to the depth averaged flow parameters at that point.

Above assumptions make the sediment continuity equation, sufficient, to describe sediment-flow. The equation for the conservation of momentum for the sediment is implicitly represented by the relationship between the sediment discharge and depth averaged flow parameters. In this section, governing equations for water and sediment are presented.

2.1 Water Flow

The two-dimensional unsteady gradually varied flow equations in open channels are (Lai 1977, Jimenez 1987, Chaudhry 1993):

Continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} + \frac{\partial(vh)}{\partial y} = 0 \quad (1)$$

Momentum equation in x-direction:

$$\frac{\partial(uh)}{\partial t} + \frac{\partial(u^2 h + \frac{gh^2}{2})}{\partial x} + \frac{\partial(uvh)}{\partial y} = gh(S_{bx} - S_{fx}) \quad (2)$$

Momentum equation in y-direction:

$$\frac{\partial(vh)}{\partial t} + \frac{\partial(uvh)}{\partial x} + \frac{\partial(v^2 h + \frac{gh^2}{2})}{\partial y} = gh(S_{by} - S_{fy}) \quad (3)$$

In the above equations (Eqs. 1-3), h is the flow depth, u and v are the depth averaged velocities in x - and y - directions respectively, g is the acceleration due to gravity, and S_{bx} and S_{by} are bed slopes in x - and y - directions respectively, x - and y - are co-ordinate axes in longitudinal and lateral directions and t is the time.

The friction slopes are calculated using the following equations

$$S_{\#} = \frac{n^2 u \sqrt{(u^2 + v^2)}}{h^{\frac{4}{3}}} \quad (4)$$

$$S_{\#} = \frac{n^2 v \sqrt{(u^2 + v^2)}}{h^{\frac{4}{3}}} \quad (5)$$

in which, n is Manning's roughness coefficient. Although n is in general a complicated function of flow depth, bottom roughness, bed slope, discharge and bed forms, a constant value is generally assumed.

Equations (1) - (3) are obtained by depth averaging the three-dimensional equations and making the following assumptions.

- (1) Acceleration in the vertical direction is negligible.
- (2) Velocity distribution is uniform over the flow depth.
- (3) Bottom shear stress is dominant and all other shear stresses are negligible.
- (4) Friction losses using steady state formula are valid for unsteady state.
- (5) Channel bottom slope is small.
- (6) Flow is not spatially varied.

Above assumptions are valid for most of the gradually varied flow situations. However, the governing equations do not account for the effective stresses which arise due to (i) laminar viscous stresses, (ii) turbulent stresses, and, (iii) stresses due to depth averaging.

Extra turbulent stress-like terms appear while depth averaging the momentum equations because of the non-uniformity of the velocity in the vertical direction. Based on experiments in the laboratory, Odgaard and Bergs (1988) have shown that the error introduced by uniform velocity assumption is negligible. Flokstra (1977) also showed that away from the walls the effective stresses are dominated by the bottom stress. However, it should be noted that these effective stresses should be considered while simulating circulating flows (Flokstra 1977). Therefore, the above equations are not valid where flow separation occurs.

2.2 Sediment Flow

As discussed earlier, only sediment continuity equation and a sediment discharge - water flow relationship are required for completely representing the sediment flow. In cartesian co-ordinates, the sediment continuity equation is given by

$$\frac{\partial Z}{\partial t} + \frac{1}{1-\lambda} \left(\frac{\partial(q_{xx})}{\partial x} + \frac{\partial(q_{yy})}{\partial y} \right) = 0 \quad (6)$$

Where, Z is the bed elevation, q_{xx} and q_{yy} are the sediment discharges per unit length in x - and y - directions, respectively and, λ is the porosity of the bed material. g_{xx} and g_{yy} depend on the flow parameters at that point. Several relations are available for estimating these quantities. Some common names in sediment discharge formulas for non-cohesive sediments are: DuBoys, Schoklitsch, Shields, Meyer-Peter, Meyer-Peter-Muller, Einstein-Brown, Laursen, Colby, Bagnold, Blench, Engelund-Hansen, Inglis-Lacey, Toffaleti, Graf, Shen-Hung, Ackers-White, Yang, Maddock, Engelund-Fredsoe, Karim, Brownlie, Rijn. These relations are well documented elsewhere (Graf 1971, Jansen et al.1979, Richards 1982, and Garde and Rangaraju 1985). Nakato (1990) presented required input data and their output for many of the formulas. Yang and Molinas (1982) evaluated six important sediment discharge formulas for the case of five rivers. These formulas are however, site-specific. Application of any formula needs verification before use. As suggested by Vanoni (1975) and Jansen et al. (1979) many of these equations can be represented in the following functional form.

$$\chi = \mathfrak{F}(Y) \quad (7)$$

where, χ is a transport parameter and Y is a flow parameter. The transport parameter depends on the sediment discharge and grain properties, while the flow parameter depends on the flow properties and bed characteristics.

2.3 Junctions

In branched flow, the conditions at junctions are to be satisfied in addition to the governing equations mentioned above. These are;

Continuity Equation:

$$\sum Q_i - \sum Q_o = \frac{\partial S}{\partial t} \quad (8)$$

Energy Equation:

$$h_1 + Z_1 + \frac{Q_1^2}{2A_1g} = h_2 + Z_2 + \frac{Q_2^2}{2A_2g} + E_L \quad (9)$$

In the above equations, Q is the discharge, S is the storage at the junction and E_L is the energy loss between the sections. It may be noted that the cross sectional areas, A_1 and A_2 , are perpendicular to the flow at those sections.

2.4 one-dimensional Equations

The one-dimensional equations can be obtained from the two-dimensional equations by averaging them for any cross-section and assuming, that the velocity, bed level and flow depth, do not vary across the width. These are given as below:

Continuity equation for water:

$$\frac{\partial Bh}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (10)$$

Momentum equation for water:

$$\frac{\partial Q}{\partial t} + \frac{\partial \left(\frac{Q^2}{Bh} \right)}{\partial x} + gBh \frac{\partial h}{\partial x} = gBh(S_b - S_f) \quad (11)$$

Sediment Continuity equation:

$$\frac{\partial [ZB(1 - \lambda)]}{\partial t} + \frac{\partial BQ_s}{\partial x} = 0 \quad (12)$$

In the above equations, Q_s = sediment discharge, B = channel width.

In addition to the above equations, equations for sediment transport and bed resistance have to be used and the equations for the junctions are also to be satisfied.

3.0 NUMERICAL SOLUTION

The governing equations for unsteady flow of water and sediment transport as discussed in section 2 constitute a set of non-linear hyperbolic partial differential equations (Lyn 1987). Analytical solutions for these equations are available only for idealized cases. Therefore, they are solved numerically. In this section, numerical schemes are presented for the solution of these equations.

The governing equations should be solved simultaneously. Complete unsteady equations should be considered for accuracy. However, the following assumption makes the model computationally efficient. Bed level changes occur at a much slower speed than the flow changes and unsteady terms in water flow equations may not be important while simulating the bed transients (De Vries 1965). Therefore, a quasi-steady approach where the flow parameters for water are constant during a small period of bed elevation change due to sediment flow may be used. The water flow equations and the sediment continuity equation are solved separately in an uncoupled model. The strategy for solution (Fig. 3) is to first compute the steady flow parameters for computing the sediment discharge values at different sections. These sediment discharge values are then used in sediment continuity equation for computing the bed level changes. After solving the sediment continuity equation, the flow computation is repeated with the new bed configuration and the procedure is continued until the required time level is reached. In the following subsection, various modules of the flow chart are described.

3.1 Flow Model

Flow model for a branched flow is performed by numerically solving continuity equation, momentum equation and equations at junctions. In the momentum equation, friction slope may be calculated using a suitable bed friction formula (e.g. Manning equation, Darcy-Weisbach equation etc.). One may consider one- or two-dimensional equations depending on the problem under consideration. A suitable numerical scheme is then adopted to solve the partial differential equations. Different methods include Finite-Difference Method, Finite-Element Method, Finite-Volume Method, Spectral Methods and Method of Characteristics. For the analysis of flows in open channels, Finite-Difference Methods are well developed and a lot of numerical experimentation on various aspects of the method has already been tested. Numerical methods described by Mahmood and Yevjevich (1975), Chaudhry (1981) and Chaudhry(1993) are very well suited to solve shallow water unsteady flow equations. Advanced methods are also available in literature for the above purpose (Savic and Holly 1991, Garcia-Navaro et al. 1992, Molls and Molls 1998,). Methods using advanced equations (other than Saint Venant equations) are also found in the literature (Tome and McKee 1994, Khan and Steffler 1996, Tsai and Yue 1996, Rudman 1997). The numerical scheme should be stable and convergent. The effect of grid size and time step size should be tested. Before applying the model to the actual problem, the performance of the model should be verified against known data for similar situations. In case of steady flow equations, the solution procedure becomes easy and very accurate numerical integration methods are available (Press et al. 1993). For the solution of the energy equation at the junctions, a suitable method to solve non-linear algebraic equations may be used.

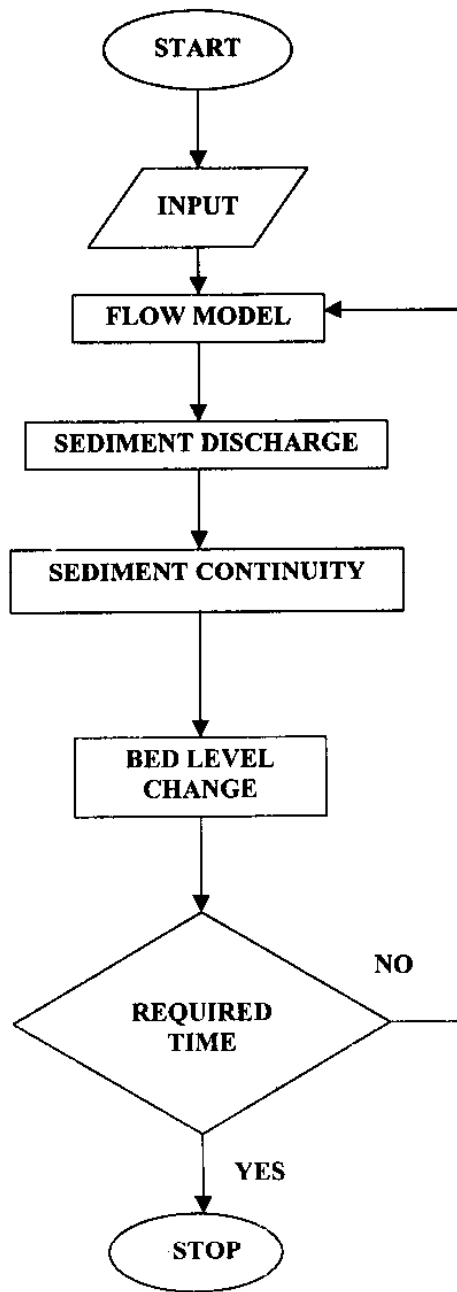


Fig. 3 Flow Chart for Quasi-Steady Uncoupled Model

3.1.1 Boundary Conditions

The system of equations is closed by implementing the boundary conditions. Implementation of boundary conditions depends on type of flow (subcritical or super critical). There are two types of boundaries in a branched flow. An open boundary could be either an inflow or an outflow and the junctions are treated as internal boundaries. In an inflow boundary, a negative characteristic equation is solved using the prescribed time history of discharge. Similarly, at an outlet, a positive characteristic equation and a known stage-discharge relationship are solved. In a junction, continuity equation and energy equation between the neighboring sections are satisfied.

3.2 Sediment Discharge

The sediment routing is performed by two parts, i.e. (1) computation of sediment discharge using a suitable formula, and, (2) numerical solution of the sediment continuity equation. For time-dependent and non-equilibrium sediment transport, the bed material is divided into several parts depending on their particle size. There are many sediment discharge relations available. It may be noted that none of these is suitable for all purposes. Therefore, a comparative study among the formulas should be performed and the closest one should be used. A correction may be applied to the estimated sediment discharge based on formulae. This correction is due to availability, sorting and diffusion of sediment particles.

3.2.1 Boundary Conditions for Sediment Flow

The rate of sediment inflow into the flow domain is given by the upstream boundary condition. If the rate is known, it may be prescribed. When unknown, the upstream boundary is extended further and the bed is assumed unchanged at that place. An extrapolation boundary condition for the downstream boundary may be used.

3.3 Sediment Continuity

Sediment continuity equation is also a non-linear partial differential equation. Therefore, it may be solved numerically. The discretization method (conversion of partial differential equation to algebraic equation) can be different from that used in the flow equation. Generally, the time step used in this equation is larger than that used in the solution of flow equation. However, in case of a coupled model, the flow equations and the sediment continuity equation have to be solved simultaneously. Therefore, a common discretization method and the same time step may be used.

3.4 Bed Level Change

Change in the cross sectional area is obtained in the numerical solution of the sediment continuity equation. This change in the cross section is applied to bed and banks. First, the direction and rate of width adjustment, and then the bed level changes are calculated. If the banks are stable, only the bed level changes are calculated.

3.4.1 Width Change

For a given time step, it is assumed that the spatial distribution of the stream power along the channel reach moves towards uniformity. A reduction in width at a cross section is due to decrease in energy gradient at the section and an increase in width is due to increase in the energy gradient. Therefore, based on the energy gradient of the section with respect to weighted average of the neighboring sections, width at the section is adjusted. Width changes in alluvial rivers are characterized by widening during channel

bed aggradation and reduction in width at the time of degradation (Chang, 1988). For a time step, the magnitude of change in width depends on the sediment discharge rate, bank configuration, and bank stability. An increase in width is due to sediment removal along the banks. For the erosion of banks, a factor (bank erodibility factor) is used. The factor varies from zero (for non-erodible banks) to one (for highly erodible banks) and its value should be determined by calibration. A decrease in the channel width is due to sediment deposition along the bank.

After the calculation of change in bank widths, bed level is adjusted. Total change in cross sectional area is the sum of change in area due to width and due to bed level. Thus, the aggradation or degradation of the bed can be found out. Methods are also available to calculate the change in bed level along the width (Chang 1988).

Quantitative time-dependent models of width adjustment that are currently available are presented in Table 1 (ASCE Task Committee, 1998). These models can be divided into two broad approaches: (a) those based on extremal hypothesis; and (2) those based on the geofluvial approach. The former have been used in engineering practice more frequently than the latter.

Table 1: List of mathematical models for width adjustment

Model/Authors	Year	Category
Darby and Thorne	1996	Geofluvial, Cohesive bank
CCHEBank(Li and Wang)	1993	Geofluvial, Noncohesive bank
Kovacs and Parker	1994	Geofluvial, Noncohesive bank
Wiele	1992	Geofluvial, Noncohesive bank
RIPA (Mosselman)	1992	Geofluvial, Cohesive bank
Simon et al.	1991	Geofluvial, Cohesive bank
Pizzuto	1990	Geofluvial, Noncohesive bank
STREAM2 (Borah and Bordoloi)	1989	Geofluvial, Cohesive bank
GSTARS (Yang et al.)	1988	Extremal hypothesis
FLUVIAL-12 (Chang)	1988	Extremal hypothesis
Alonso and Combs	1986	Geofluvial, Cohesive bank
WIDTH (Osman)	1985	Geofluvial, Cohesive bank

3.5 Calibration of the Model

The success of a mathematical model greatly depends on the equations used to account for the actual physical processes, implementation of boundary conditions and numerical method applied. Before applying the model to actual situation, it must be tested for known results and various model parameters should be calibrated. Important items requiring calibration are Manning roughness coefficient, sediment discharge relation, bank erodibility factor, and bed erodibility factor. Numerical parameters such as step size and time steps should also be tested. The limitations of the model should be clearly mentioned.

3.6 Data requirement

The following data are required to compute the flow in an island.

Time variation of discharge in the main channel,
Cross-section details of main and branched channels,
Stage - discharge relationship for the downstream main channel,
Bed slope for all the channels,
Roughness characteristics of all the channels, and,
Grain size distribution of the flowing sediment.

3.7 Modelling of a simple island

In the above sub-sections, methodology to model flow in a single reach is described. For an island flow, the junctions are treated as an internal boundary. The continuity equation, the energy equations and the characteristic equations depending on the case of a upstream or downstream boundary, are to be satisfied for the junctions. A proposed model is presented in the Appendix. However, in the absence of observed data, the model has not been validated. The performance of the model in terms of its computational efficiency and robustness is subjected to further study.

4.0 COMPUTER CODES IN SEDIMENT TRANSPORT

4.1 one-dimensional Models

BHALLAMUDI AND CHAUDHRY: Bhallamudi and Chaudhry (1991) developed an unsteady water and sediment routing model to simulate aggradation and degradation of channel bottom. They solve the Saint Venant Equation and sediment mass conservation equation simultaneously using the MacCormack explicit Finite-Difference method. The model can handle shocks and discontinuities in the solutions of these equations without iterations. It uses *Manning's n* for friction and power functions of unit discharge and depth for calculating sediment discharge capacity.

CHARIMA: It is an extended version of IALLVIAL and was developed by Holley et al. (1990). It solves flow and sediment routing in unsteady multiply-connected fluvial channels with reverse flows. It includes TLTM, the Ackers-White, and Engelund-Hansen formulas, and a site specific power law for non-cohesive sediment discharge calculations. The model also simulates cohesive sediment routing. Jain and Park (1989) have conducted a similar modeling with the use of the Karim's coupled friction factor and sediment discharge relationships.

FLUVIAL 11: FLUVIAL 11 is an unsteady, one-dimensional, finite-difference flood and sediment routing model formulated in a curvilinear coordinate system (Chang 1984). The model calculates inter related changes in channel bed profile, width, and lateral migration in channel bends. The energy slope is divided into the longitudinal energy gradient and the transverse energy gradient as a result of a secondary current existing in a curved channel. The model uses the Yang, Engelund-Hansen, Graf, Parker et al. (1982) and Ackers-White sediment discharge formulas.

HEC-2SR: It is a combination of water routing model and sediment routing model. The water routing model is HEC-2 and was developed by Hydraulic Engineering Center (1982). The sediment routing model was developed by Simons et al. 1980. HEC-2SR uses a step backwater computation method for water routing. It solves the Exner equation for sediment routing. The model uses Meyer-Peter-Muller formula for the bed load and the Einstein method to calculate suspended sediment capacity.

HEC-6:HEC-6 was developed by the hydrologic Engineering Center in 1977. It is one of the widely used codes. It uses one-dimensional flow and sediment equations to simulate riverbed profile changes over years. Flow is assumed steady and the gvf equation is solved. A series of steady flow events can be connected to represent long-term continuous flow. The model does not use a stage -discharge predictor directly to account for the effect of bed form on hydraulic roughness. It separates energy losses into friction loss and form loss (due to channel expansion/contraction). Manning's roughness coefficient for bed roughness is used in the model to express friction loss. For sediment transport, the model uses the Exner equation (Eq. 5.4) with sediment discharge formulas. A user can select one of the following sediment discharge formulas: Toffaleti, Modified Laursen, Yang, DuBoys, Ackers-White, Colby, Combination of Toffaleti and Schoklitsch, Meyer-Peter-Muller, Combination of Toffaleti and Meyer-Peter-Muller, Partheniades-Krone, and a user specified relationship.

IALLVIAL: IALLVIAL is a quasi-steady, finite-difference flow and sediment routing model (Karim et al. 1987). It predicts water routing by step back water method. Exner equation is solved for the sediment mass balance. It includes an iterative coupled sediment discharge and friction factor predictor (TLTM). TLTM was developed by Karim and Kennedy (1981).

KUWASER: KUWASER was developed by Simons and his associates (NRC 1983) to simulate steady state, one-dimensional flow and sediment transport. The sediment discharge per unit width is expressed as a power function of mean flow velocity and depth and is site specific.

ONED3X: The US Geological Survey developed a series of computer codes collectively called ONED3X by solving fully coupled unsteady, one-dimensional, flow and sediment equations by multi mode method of characteristics (Lai 1988). The sediment concentration is assumed to be a power function of velocity and water depth, and its functionality is site specific. The codes use the Manning or Chezy equation for friction.

STARS: STARS is a steady, one-dimensional, water and sediment routing model (Molinas 1983). Its unique feature is the use of stream tubes to divide each cross section into multiple equal discharge sections. This allows lateral variation of flow and sediment movements. Thus, the model can simulate simultaneous erosion and deposition within the same cross section. It uses the Meyer-Peter-Muller, Einstein, Engelund-Hansen, Toffaleti, Yang, and Ackers-White sediment discharge formulas. Manning's n is used for friction. Use of the stream tubes was further extended by Molinas and Yang (1986) to develop GSTARS to handle one-dimensional, semi- two and three dimensional cases of supercritical, critical and subcritical flows. GSTARS uses the Yang, Engelund-Hansen, and Ackers-White sediment discharge formulas. It uses Manning, Darcy-Weisbach, or Chezy equations to determine the energy loss along the river reach.

TODAM: TODAM is an unsteady, one-dimensional, finite element, sediment and contaminant transport code, without water routing predictive capabilities (Onishi et al. 1982). It solves for distributions of cohesive sediment, non-cohesive sediment, contaminants, (eg. toxic chemicals, heavy metals and radio nuclides) attached to the sediments, and dissolved contaminants in water and bed, with the output of a hydrodynamic model. TODAM includes sediment contaminant interactions and transport, deposition, and erosion of sediment and sediment sorbed contaminant, as well as chemical and biological degradation/decay of contaminants. Changes of bed conditions are calculated by bookkeeping of the changes occurring during the simulation. TODAM is applicable to rivers and well-mixed estuaries.

UUWSR: It was developed by Tuci et al. (NRC 1983) as an unsteady, one-dimensional, uncoupled water, and sediment routing model. Treatments of the friction factor and sediment discharge capacity calculations are similar to those in KUWASER.

4.2 Two-Dimensional Models

There are far fewer true two-dimensional models for sediment transport than one-dimensional models. The two-dimensional models generally solve the Reynolds form of the Navier-Stokes equation, in place of the Saint-Venant equations. They also solve the

advection-diffusion equation as often as they solve a two-dimensional version of the Exner equation. The following are examples of the two-dimensional models.

FETRA: FETRA is an unsteady, two-dimensional, finite-element sediment and contaminant transport code (Onishi 1981). It does not have its own hydrodynamic calculation. Its input requirements are compatible with output from the finite-element hydrodynamic codes RMA-2 and CAFE. It simulates transport, deposition, and re-suspension of sediments, dissolved contaminants, and sediment-sorbed contaminants in water and bed, along with their interactions. FETRA is applicable to rivers, estuaries, and coastal waters. It uses DuBoys' formula for non-cohesive sediment without surface waves. When wave effects are important, Liang and Wang formula and Komar formula are used to obtain the wave induced sediment discharge. In case of cohesive sediments, Partheniades and Krone's formulas are used.

ODGAARD: Odgaard (1989) developed a steady, two-dimensional hydrodynamic and sediment transport code. It uses orthogonal co-ordinate system to solve meandering flow and associated meandering development and sediment transport. Since depth-averaged flow cannot reflect the helical motion caused by centrifugal acceleration acting on the flow, Odgaard assumed vertical distribution of the longitudinal and lateral velocities. By linearizing velocities, he then cast the momentum equations into two variables, lateral velocity gradient and lateral bed slope along the center line. The module uses Darcy-Weisbach equation for friction. It also assumes a power law for the longitudinal sediment discharge, while using Ashida and Michiue's (1972) relation to correlate the lateral sediment discharge to the longitudinal discharge.

SEDIMENT-4H: It is very similar to TABS-2. It is based on one- and two-dimensional hydrodynamic and sediment transport formulation (Ariathurai 1980).

SERATRA: SERATRA is an unsteady, finite-element sediment and contaminant transport code. It is a two-dimensional version of TODAM. It was developed by Onishi et al (1982).

SHIMIZU & ITAKURA: Shimizu and Itakura (1989) developed a steady, two-dimensional hydrodynamic and sediment transport model in a orthogonal curvilinear coordinate system to handle symmetric and unsymmetrical meandering channel flow. The model solves the Navier-Stokes equation and Exner equation. It uses Manning equation for bed friction. For the sediment discharge calculation, it uses the Mayer-Peter-Muller formula for longitudinal sediment load and Hasegawa equation for lateral sediment load.

TABS-2: The Waterways Experiment Station developed several unsteady, two-dimensional, finite-element hydrodynamic and sediment transport computer codes collectively called TABS-2. These codes are applicable to rivers, reservoirs and estuaries. The main components of TABS-2 are the hydrodynamic component (RMA-2V), the sediment transport component (STUDH), and the water quality component (RMA-4). RMA-2V solves the Reynolds equation. It does not take into account the interaction between bed form and friction factor. STUDH can compute both cohesive and non-cohesive sediment transport. It uses the Ackers-White sediment formula for bed material transport capacity. Changes in bed conditions are handled by bookkeeping.

TWODSR: TWODSR is an unsteady, two-dimensional, uncoupled, finite-difference water and sediment-routing model. It uses the Reynolds equations with the continuity equation to simulate hydrodynamics. For sediment transport, it uses a two-dimensional expression of the Exner equation with sediment transport capacity expressed as a power function of flow discharge. For bed friction, it uses Manning equation or Chezy equation.

4.3 Three-Dimensional Models

FLESCOT: FLESCOT (Onishi et al. 1985) is a sediment contaminant version of the general hydrodynamic transport code TEMPEST (Onishi et al. 1985). It is an unsteady, three-dimensional, finite-difference code and simulates flow, turbulence, water temperature, salinity, sediment, dissolved contaminants, and sediment sorbed contaminants for rivers, estuaries, lakes, coastal waters, and oceans. The code can run with or without the use of hydrostatic pressure assumption. It uses three-dimensional forms of equations for sediment transport. The code uses Manning equation, Chezy equation or Darcy-Weisbach equation for bed friction. It uses DuBoys' formula for non-cohesive sediment and Partheniades formula for cohesive sediment. Grant's non-linear, wave enhanced bottom shear stress formula (Grant and Madsen 1979) is built into the code to calculate wave -enhanced sediment transport (Onishi et al. 1993), as one of the options.

SHENG: Sheng (1993) developed an unsteady, three-dimensional, finite-difference code to simulate flow, turbulence, salinity, water temperature, and sediment and water quality parameters. It is applicable to rivers, lakes, estuaries, coastal waters, and oceans. The code uses a two-mode hydrodynamic calculation with internal and external modes. For the external mode, it calculates water surface elevations by solving depth-averaged hydrodynamic equations with a small time step. With the calculated water surface, the internal mode then calculates three-dimensional velocity distributions with a much larger time step. This approach is very useful for long-term simulations.

5.0 CONCLUSION

In this work, numerical modeling of branched open channel flows with alluvial conditions was presented. Governing equations for water flow, sediment-flow and flow at junctions were presented. Numerical methods to solve these equations for engineering applications were described. Various computer codes suitable for open channel flows in alluvial conditions were mentioned. Based on the literature survey, a One Dimensional model, for simple-island flow in alluvial conditions, using quasi-steady flow assumption and uncoupled approach was proposed. The following important conclusions were drawn out of the present study.

1. Highly accurate equations are available for rigid bed flows. The mechanics of flow in alluvial conditions is not well understood.
2. There are a number of numerical models available for flows in alluvial conditions. Suitability of a model depends mostly on the sediment discharge relation used.
3. A large number of sediment discharge relations are available. These are either empirical or semi-empirical in nature and therefore, cannot be used universally. Their successful performance (accurate predictions) greatly depends on the similarity to conditions of their origination.
4. In a mathematical model for alluvial streams, the bed roughness characteristics should be calibrated.
5. The proposed model must be verified for measured data before its application to field situation.
6. Calibration of parameters (bed roughness coefficients and sediment parameters) should be performed before its use.

The following suggestions are made for future research.

1. Calibration of the proposed model (see Appendix).
2. Development of sediment discharge based on flow velocity distribution in a vertical plane (not as a function of depth averaged velocity).
3. Two-dimensional modeling near junctions taking secondary flows into account.
4. Laboratory study (physical modeling) of a branched flow in alluvial conditions.
5. Erosion of banks and aggradation/degradation of beds with help of a parametric study.

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Appendix

MODELING OF A SIMPLE ISLAND

In this appendix, a model is proposed to study the case of a simple island with alluvial conditions. The model uses a quasi-steady uncoupled approach and an explicit finite-difference formulation for one-dimensional equations. The model is described in the following paragraphs.

Governing Equations

Continuity Equation for Water:

$$\frac{\partial Bh}{\partial t} + \frac{\partial VBh}{\partial x} = 0 \quad (A-1)$$

Momentum Equation for Water:

$$\frac{\partial Q}{\partial t} + \frac{\partial(V^2 Bh)}{\partial x} + gBh \frac{\partial h}{\partial x} = gBh(S_b - S_f) \quad (A-2)$$

Sediment Continuity Equation:

$$\frac{\partial[ZB(1-\lambda)]}{\partial t} + \frac{\partial BG}{\partial x} = 0 \quad (A-3)$$

Friction Formula:

$$S_f = \frac{n^2 V^2}{h^{\frac{4}{3}}} \quad (A-4)$$

Sediment Discharge Equation:

$$G = a(V)^b \quad (A-5)$$

Continuity Equation at the Junction:

$$\sum Q_i - \sum Q_o = 0 \quad (A-6)$$

Energy Equation at the Junction:

$$h_1 + Z_1 + \frac{V_1^2}{2g} = h_2 + Z_2 + \frac{V_2^2}{2g} + E_L \quad (A-7)$$

In the above equations, V is the depth averaged flow velocity, and, a and b are the constants used in the sediment discharge formula.

Numerical Solution

The simple island consists of four channels and each channel is divided into a number of reaches (Fig. A-1). Thus, any section is represented by $x_{i,j}$, where, i and j indicate the channel and node respectively. For example, $x_{2,5}$ is the fifth node in the second channel. Similarly, velocity, flow depth, bed level and cross sectional area are represented by double subscripted variables. As the flow is transient, all flow variables are superscripted with time level. Thus, $h'_{i,j}$ represents the flow depth of channel i in j^{th} section at time t .

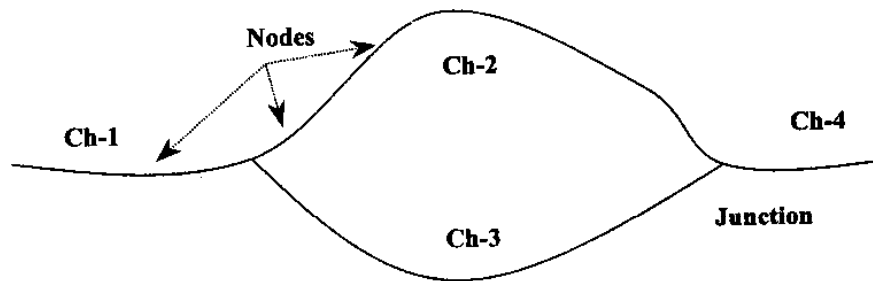


Fig. A-1: A Simple Island

Input

The input values are; (1) Channel cross sections at selected places for all the four channels, (2) Bed slopes at different reaches, (3) Lengths of each channel, (4) Number of reaches in each channel, (5) Manning n for different reaches, (6) Courant number for stability criteria (calculation of time step), (7) Acceleration due to gravity, (8) Sediment parameters a and b , (9) Time of computation, (10) Stage-discharge relationship for downstream end, and (11) Time history of discharge at upstream end.

Computational Grid

Based on the channel length and number of reaches, the computational grid size is computed for each channel.

FLOW ROUTING

Time Step: The computational time-step for the flow equations is determined using Courant condition of stability. The computed time step may be different for different channels. The minimum of all the values is used. Time step is calculated in the beginning of each computational cycle.

St-Venant: The flow equations, Eqs. A-1 and A-2, (Saint Venant Equations) are solved using MacCormack scheme (Chaudhry 1981, Chaudhry 1993). This scheme is second order accurate and is suitable for both sub- and super-critical flow situations. It is capable

of capturing shocks also. The friction slope term, S_f , in Eq. A-2 is evaluated by using Eq. A-4.

Boundary Conditions: All flow variables are known at the start of computation (initial conditions). For a new time level, these are computed using the numerical solution of the governing equations only at the internal nodes. The flow variables at the boundaries are not determined using the numerical solution of the governing equations. The following boundary conditions are used to calculate the variables.

<u>Upstream of Ch-1:</u>	Negative characteristics equation and inflow hydrograph.
<u>Downstream of Ch-4:</u>	Positive characteristics equation and stage-discharge relationship.
Junction-1:	Continuity for Q using Eq.A-6 ($Q_1=Q_2+Q_3$) Energy equation for last node of Ch-1 and first node of Ch-2 Energy equation for last node of Ch-1 and first node of Ch-3 Positive characteristic equation for last node of Ch-1 Negative characteristic equation for first node of Ch-2 Negative characteristic equation for first node of Ch-3
Junction-2:	Continuity for Q ($Q_2+Q_3=Q_4$) Energy equation for last node of Ch-2 and first node of Ch-4 Energy equation for last node of Ch-3 and first node of Ch-4 Positive characteristic equation for last node of Ch-2 Positive characteristic equation for last node of Ch-3 Negative characteristic equation for first node of Ch-4

The flow model is executed until it attains nearly steady state and then the control is passed to the sediment routing model.

SEDIMENT ROUTING

Sediment Discharge: After the computation of flow variables, sediment discharge is estimated by using Eq.A-5. This relation is only valid for sediments of uniform particle size. It may be noted here that this may be replaced by a better formula if required. This is used here only due to its simple form.

Sediment Continuity: sediment continuity equation, Eq. 15, is solved by a first order scheme. The discretization is performed by backward finite-difference. Generally, the time step used for sediment continuity [$O(1000s)$] is much larger than that used in the flow routing [$O(1 s)$]. The value of time step is decided by numerical experimentation using the criterion that the end-result is not affected by the highest value. The scheme being backward finite-difference, only the boundary condition at the upstream end has to be prescribed. In this model, it is assumed that the initial condition remains same.

Bed Width Modification: The change in cross sectional area is calculated by solving the sediment continuity equation. This change in cross section is partly due to change in width and partly due to change in elevation. First, the direction of width adjustment is

found out by comparing the friction slope, S_f , with that of the weighted friction slope of the neighbors, \underline{S}_f

If $S_f > (\underline{S}_f)$ then Width is decreased

If $S_f < (\underline{S}_f)$ then Width is increased

If $S_f = (\underline{S}_f)$ then Width is not changed

The magnitude of change in width is calculated by multiplying a factor (0 to 1) with the sediment discharge rate.

Bed Level Modification: After calculating the change in bed width, the bed elevation is adjusted such that the change in area remains constant. Thus, the aggradation and/or the degradation of the channel bed can be known. The modified bed slopes are calculated based on the modified values of bed elevation

The whole of the computation, starting from the flow routing is repeated till the required time level is attained.

Limitations

The present model is not verified against measured data. Therefore, its performance is not known. However, based on the literature work following limitations should be borne in mind.

1. Flow is one-dimensional.
2. Bed roughness coefficient is constant.
3. Estimation of sediment transport parameters (a and b in Eq. A-4).
4. Estimation of energy loss in the energy equation (Eq. A-7), at the junctions.
5. Use of a first order scheme for sediment continuity.

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