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ANALYSIS OF FLOW TO A MULTI - AQUIFER WELL : A NUMERICAL APPROACH

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Abstract

Existence of multi-aquifer system are common in a sedimentary basin. A well drawing water from more than one aquifers that have good quality of water, increases the yield of the well and in most of the cases is economical. In most of the studies done so far, pumping from only one layer has been considered. In very few analytical studies, pumping from more than one aquifer has been dealt. The available analytical solutions can not take into account the complex boundary conditions and spatial variations in the values of the parameters within the flow domain. For these, numerical models are required. However, three dimensional groundwater flow models can not accommodate a well taping more than one aquifer of a multi-aquifer system.

In the present study, unsteady flow towards a fully penetrating well screened in both the aquifers of a two-aquifer system separated by an aquitard has been analyzed using numerical approach. The aquifers are considered to has different hydraulic characteristics. A three dimensional groundwater flow model developed by USGS has been used to generate the unit pulse coefficients. Utilizing these. unit pulse coefficients, methodology has been developed for obtaining the contribution of each aquifer and drawdown in the aquifer. The results using the methodology have been compared to those available from analytical study. Therefore, using the present methodology, a three dimensional groundwater flow model or MODFLOW can be successfully used for modelling flow to a multi-aquifer well incorporating the spatial variations in the parameters within the flow domain.

Contents

1.0 INTRODUCTION

Groundwater abstraction being mainly through pumping well, the hydraulics of flow towards wells has been the prime interest of civil engineers and hydrologists dealing with groundwater hydrology since the beginning of this century. Dupuit(1963) analyzed steady state flow towards a well. Considerable research has been done on transient flow towards wells after the pioneering work of Theis (1935).

In a sedimentary basin, multi-aquifer systems are commonly encountered. In a multi-aquifer system a number of aquifer layers are separated by confining layers. Multi-aquifer systems can be classified into three groups (Kruseman and DeRidder, 1990), viz., i) two or more aquifer layers separated by aquicludes, ii) two or more aquifer layers separated by aquitards, iii) two or more aquifer layers that allow unrestricted cross flow through their interfaces. The yield of the well can be increased by taping more than one aquifer of a multi-layered aquifer system. When a well taping more than one aquifer of a multi-aquifer system, is pumped, contributions to the pumped discharge from aquifer layers are different. The extent of differences among the contributions from different layers depend upon the hydraulic properties of different aquifers. The contribution from each aquifer layer to pumped discharge is required for the complete analysis. Flow towards a well tapping more than one aquifer of a multi-aquifer system can not truly be modelled by even three dimensional groundwater flow models, like one by USGS (MacDonald and Harbaugh, 1986).

In the present study, unsteady flow towards a fully penetrating well screened in both the aquifers of a two-aquifer system separated by an aquitard has been analyzed using numerical approach. The aquifers are considered to has different hydraulic characteristics. A methodology has been developed which enables the use of MODFLOW for realistic modelling of flow to a multi-aquifer well. The results obtained using the methodology are found in conformity to those available from analytical studies.

2.0 REVIEW

Skol(1963) has analyzed steady state drawdowns due to a non pumping well perforated in more than one aquifer-layer. He observed that the drawdown in the well is affected by each aquifer in proportion to the transmissivity of that aquifer. Papadopulos (1966) has derived solutions for the transient flow towards a well open to two aquifers of different hydraulic properties and separated by aquiclude. He found that the exact solutions to the problem are intractable for numerical calculations while the asymptotic solutions are amenable to easy computations and are accurate enough for practical applications. Hantush(1967), and Neuman and Witherspoon(1969) have analyzed the flow towards a multi-aquifer well which taps only one of the aquifers.

Nautiyal (1984) analyzed multi-aquifer well problem using discrete kernel approach for i) unsteady flow to a well tapping aquifers separated by aquicludes ii) Unsteady flow to a well tapping two aquifers separated by an aquitard. He considered pumping from all the aquifer layers. The main conclusions are,

1. If the aquifers have equal diffusivity values, the contributions by each of the aquifers through the well screens during pumping at a constant rate are independent of time and proportional to the respective transmissivity values. In such a case, there is no exchange of flow through the intervening aquitard irrespective of the magnitude of leakage factor and the drawdowns at any section in both the aquifer are same.

2. Aquifer whose hydraulic diffusivity is lower its contribution to well discharge through the screen decreases as the pumping continues. Conversely, the aquifer whose hydraulic diffusivity is higher its contribution increases as the pumping continues.

3. If the pumping continues at a constant rate for infinite period, the limit $Q_1(n)/Q_2(n)$ tends to T_1/T_2. As the leakage factor decreases, the nearly steady-state condition approaches comparatively at a shorter time.

Khader and Veerankutty(1975) used integral transform technique to solve the transient flow towards a multi-aquifer well. Mishra, Nautiyal and Chandra (1985) have presented a simple methodology for computing drawdown in a two-aquifer system separated by an aquiclude. They used discrete kernel approach to solve the problem.

Hunt (1985) analyzed the steady and unsteady flows to a well in a multi-layered aquifer system with a number of horizontal aquifers, separated by aquitards. He assumed that the well is open to a single aquifer only. However, he considered flow from all the aquifers for steady-state situation, and proposed a method to find discharge contributions from each aquifer. Javandel and Witherspoon (1983) studied the flow problem associated with a partially penetrating well in a two-layered confined aquifer with cross flow between adjacent layers. They have considered the well to be screened either in the upper layer or in the lower layer.

The groundwater flow model developed by McDonald and Harbaugh (1988) also can not accommodate a well tapping more than one aquifer of a multi-aquifer system. The analytical models summarized above are applicable for simplified boundary conditions and can not take into account the spatial variation in the aquifer parameters. Unsteady flow to a fully penetrating well tapping more than one aquifer of a multi-aquifer system can not be modelled correctly using groundwater flow models.

3.0 STATEMENT OF THE PROBLEM

A well taps full thickness of both the aquifers of a two-aquifer system. The aquifers are separated by an aquitard. The aquifer system is confined both from top and bottom. Both the aquifer are, homogeneous and isotropic and have infinite areal extent and uniform thickness. Definition sketch of the problem is given in fig. 3.1. Initially, the piezometric surfaces in both the aquifers are assumed horizontal and at same level. The well is pumped at a constant rate.

Considering the exchange of flow through aquitard, it is required to find, i) contributions to the pumped discharge from each aquifer and their variation with time; and, ii) drawdown in the well.

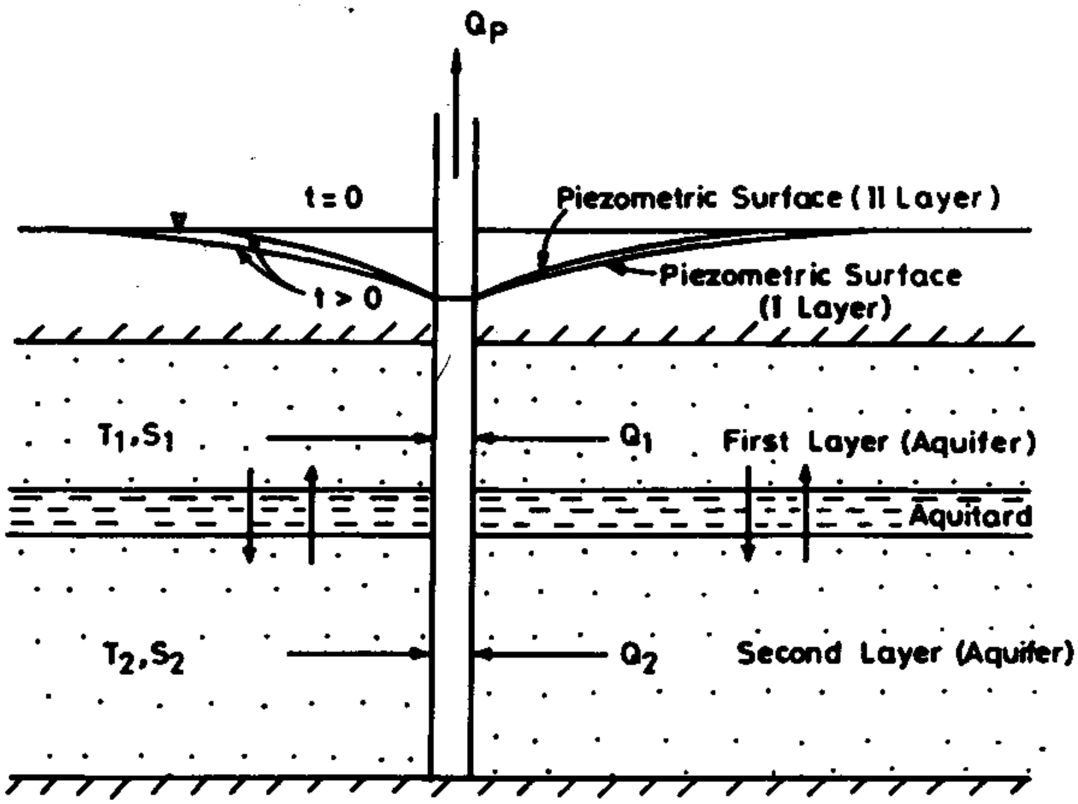

FIG. 3·1 DEFINITION SKETCH OF THE PROBLEM

4.0 METHODOLOGY

Following assumptions have been made in the analysis:

1) time parameter is discrete. Within each time step, the abstraction rates from each aquifers is a separate constant, but varies with time-step,

2) at any time, drawdown in both the aquifers at the well face are same but varies with time-step.

The partial differential equation that describes the axially symmetric, radial, unsteady flow in a confined semi-infinite aquifer is given by,

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \tag{1}$$

Where, s is the drawdown in piezometric surface, r is the radial distance from the pumped well, T and S are the transmissivity and storage coefficient of the aquifer, respectively. Solution of eq. (1) for transient pumping rate, with boundary condition as, $s(\infty,t)=0$ and initial condition as, $s(r,0)=0$, can be expressed in discrete form by the following equation (Morel-Seytoux, 1975).

$$s_n = \sum_{\gamma=1}^{n} Q(\gamma)\delta(n-\gamma+1) \tag{2}$$

Where, s_n = drawdown at the end of n^{th} time-step, $Q(\gamma)$ = pumping rate during γ^{th} time-step. $\delta(.)$ is the discrete kernel for drawdown and is expressed as,

$$\delta(m) = \frac{1}{4\pi T}\left[W\left\{\frac{r_w^2\, S}{4Tm\Delta t}\right\} - W\left\{\frac{r_w^2\, S}{4T(m-1)\Delta t}\right\}\right] \tag{3}$$

or,

$$\delta(m) = s'(m\Delta t) - s'[(m-1)\Delta t] \tag{4}$$

Where, $s'(m)$ is the drawdown in the aquifer at the end of m^{th} time step due to pumping at unit rate. Let $Q_1(n)$ and $Q_2(n)$ be the contributions at n^{th} time step from first and second aquifer respectively. Since, pumping rate is constant, the continuity equation for pumped discharge is given by,

$$Q_1(n) + Q_2(n) = Q_p \tag{5}$$

Assuming cross-flow through the aquitard, drawdown in the first layer at the end of n^{th} time step, can be expressed as,

$$s_1(n) = \sum_{\gamma=1}^{n} Q_1(\gamma)\delta_{11}(n-\gamma+1) + \sum_{\gamma=1}^{n} Q_2(\gamma)\delta_{21}(n-\gamma+1) \tag{6}$$

Where, $\delta_{11}(.)$ and $\delta_{21}(.)$ are discrete kernels for drawdowns in the first aquifer when pumping is from first aquifer and second aquifer respectively. Similar to eq. (4), the expressions for $\delta_{11}(m)$ and $\delta_{21}(m)$ can be written as,

$$\delta_{11}(m) = s'_{11}(m) - s'_{11}(m-1) \tag{7}$$

$$\delta_{21}(m) = s'_{21}(m) - s'_{21}(m-1) \tag{8}$$

Where, $s'_{11}(.)$ and $s'_{21}(.)$ are the drawdowns in the first layer due to pumping at unit rate from first layer and second layer respectively. Similarly, drawdown in the second layer at the end of n^{th} time step can be written as,

$$s_2(n) = \sum_{\gamma=1}^{n} Q_1(\gamma)\delta_{12}(n-\gamma+1) + \sum_{\gamma=1}^{n} Q_2(\gamma)\delta_{22}(n-\gamma+1) \tag{9}$$

Where, $\delta_{12}(.)$ and $\delta_{22}(.)$ are discrete kernels for drawdowns in the second aquifer when pumping is from first and second aquifer respectively. The expressions for these kernels can be written as,

$$\delta_{12}(m) = s'_{12}(m) - s'_{12}(m-1) \tag{10}$$

$$\delta_{22}(m) = s'_{22}(m) - s'_{22}(m-1) \tag{11}$$

Where, $s'_{12}(.)$ and $s'_{22}(.)$ are the drawdowns in the second aquifer due to pumping at unit rate from first aquifer and second aquifer respectively. Since, the drawdown in first and second aquifer at the well face are same at any time-step,

$$s_1(n) = s_2(n) \tag{12}$$

Substituting the values of $s_1(n)$ and $s_2(n)$ from eqs. (6) & (9) respectively, in eq. (12) and re-arranging, we get,

$$Q_1(n)\left\{\Delta\delta_1(1)\right\} + Q_2(n)\left\{\Delta\delta_2(1)\right\} =$$

$$- \sum_{\gamma=1}^{n-1} \left\{Q_1(\gamma)\Delta\delta_1(n-\gamma+1) + Q_2(\gamma)\Delta\delta_2(n-\gamma+1)\right\} \tag{13}$$

where,

$$\Delta\delta_1(.) = \delta_{11}(.) - \delta_{12}(.) \tag{14}$$

$$\Delta\delta_2(.) = \delta_{21}(.) - \delta_{22}(.) \tag{15}$$

Eqs. (5) & (13) can be expressed in matrix form as,

$$\begin{bmatrix} Q_1(n) \\ Q_2(n) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \Delta\delta_1(1) & \Delta\delta_2(1) \end{bmatrix}^{-1} \begin{bmatrix} Q_p(n) \\ -\sum_{\gamma=1}^{n-1} \left\{ Q_1(\gamma)\Delta\delta_1(n-\gamma+1) + Q_2(\gamma)\Delta\delta_2(n-\gamma+1) \right\} \end{bmatrix} \tag{16}$$

If $\delta_{11}(.)$, $\delta_{12}(.)$, $\delta_{21}(.)$, $\delta_{22}(.)$, and Q_p are known, Q_1 and Q_2 can be obtained in succession for all time steps, starting from first time step, using eq.(14), (15) & (16). With known values of $Q_1(n)$ and $Q_2(n)$, drawdown in the well at any time step can be found using either eq. (6) or eq. (9).

4.1 PROCEDURE FOR CALCULATION OF $\delta_{..}(.)$:

In order to obtain discrete kernels, i.e., $\delta_{11}(.)$, $\delta_{12}(.)$, $\delta_{21}(.)$, and $\delta_{22}(.)$, the three dimensional ground water flow model (McDonald & Harbaugh, 1988) has been used. The aquifer system has been discretized in plan by rectangular grid arrays forming a number of rows and columns. In vertical, the aquifer has been divided into three layers, the middle layer consists of aquitard and top and bottom layers are aquifers. The central grid has been considered as well and away from the grid, the spacing of the grids have been taken to be the same for both positive and negative directions of x and y axes. The first and the last rows and columns have been assumed to be no flow boundaries. Variable grid-spacing has been assumed, with finer grids near the well and coarser grids away from the well. Time span has been discretized into a number of time steps. The time unit selected for the model data is minute. Ten time steps has been taken in each stress period to insure better convergence. At the end of each simulation, piezometric heads at grid centres were obtained. In all the cases of model-runs, a convergence criterion of 0.0001 m was adopted.

The procedure to obtain the discrete pulse kernels, consists of the following steps.

1. Consider the well to be screened only in the top aquifer layer and be pumped at unit rate. Obtain the drawdown values at the well, i.e., $s'_{11}(m)$ and $s'_{12}(m)$ for top aquifer and bottom aquifer, respectively, from the model results. Here, m varies from 1 to number of stress periods,

2. Consider the well to be screened only in the bottom aquifer and be pumped at unit rate. Obtain the drawdown values at the well, i.e., $s'_{21}(m)$ and $s'_{22}(m)$ for first layer and second layer, respectively from the model results.

If $s'_{11}(.)$, $s'_{12}(.)$, $s'_{21}(.)$ and $s'_{22}(.)$ are known, all the discrete pulse kernels can be obtained making use of eqs. (7), (8), (10), & (11).

The methodology presented can easily be extended for an aquifer system consisting of more than two aquifers when a taps all the aquifers.

5.0 RESULTS AND DISCUSSIONS

Application of the methodology was taken up for four different cases of two aquifers separated by aquitard with the following values of aquifer parameters.

CASE I:
$T_1 = 0.2\ m^2/min.$ $S_1 = 0.01$
$T_2 = 0.4\ m^2/min.$ $S_2 = 0.001$
$B_1/K_1 = 1.14x10^{-5}/min.$
Drawdown obtained at 10 min. interval.

CASE II:
$T_1 = 0.2\ m^2/min.$ $S_1 = 0.01$
$T_2 = 0.4\ m^2/min.$ $S_2 = 0.001$
$B_1/K_1 = 1.14x10^{-5}/min.$
Drawdown obtained at 100 min. interval.

CASE III:
$T_1 = 0.2\ m^2/min.$ $S_1 = 0.01$
$T_2 = 0.4\ m^2/min.$ $S_2 = 0.001$
$B_1/K_1 = 1.15x10^{-4}/min.$
Drawdown obtained at 100 min. interval.

CASE IV:
$T_1 = 0.2\ m^2/min.$ $S_1 = 0.01$
$T_2 = 0.4\ m^2/min.$ $S_2 = 0.001$
$B_1/K_1 = 1.15x10^{-4}/min.$
Drawdown obtained at 1000 min. interval.

Where, T_1, T_2 are the transmissivities of first and second aquifer respectively and, S_1 and S_2 are the storage coefficient of first and second aquifer respectively. B_1 and K_1 are the thickness and hydraulic conductivity of the aquitard. For all the cases listed above, Q_p =1.0 m^3/mim. , r_w = 0.1 m.

For, each case listed above, Q_1 and Q_2 and drawdown have been obtained. The values of $r_w^2 S_1/(4T_1 t)$ and corresponding values of Q_1/Q_p are given in Table 5.1. For all the cases T_1/T_2=0.5. For cases I & II, B_1/K_1=1.14E-5 and for the cases III & IV, B_1/K_1=1.14E-4. The Cases I and II are the same except the range of $r_w^2 S_1/(4T_1 t)$ in each case. The results for these cases tabulated above show that Q_1/Q_p decreases as

Table 5.1: Variation of Q_1/Q_p with $r_w^2 S_1/(4T_1 t)$

Case I		Case II		Case III		Case IV	
$r_w^2 S_1/(4T_1 t)$	Q_1/Q_p	$r_w^2 S_1/(4T_1 t)$	Q_1/Q_p	$r_w^2 S_1/(4T_1 t)$	Q_1/Q_p	$r_w^2 S_1/(4T_1 t)$	Q_1/Q_p
1.25E-05	0.377	1.25E-06	0.362	1.25E-07	0.342	1.25E-07	0.334
6.25E-06	0.373	6.25E-07	0.355	6.25E-07	0.337	6.25E-08	0.334
4.17E-06	0.371	4.17E-07	0.350	4.17E-08	0.336	4.17E-08	0.334
3.13E-06	0.369	3.13E-07	0.347	3.13E-08	0.335	3.13E-08	0.333
2.50E-06	0.367	2.50E-07	0.345	2.50E-08	0.335	2.50E-08	0.333
2.08E-06	0.365	2.08E-07	0.344	2.08E-08	0.335	2.08E-08	0.333
1.79E-06	0.363	1.79E-07	0.343	1.79E-08	0.335	1.79E-08	0.333
1.56E-06	0.362	1.56E-07	0.342	1.56E-08	0.334	1.56E-08	0.333
1.39E-06	0.360	1.39E-07	0.341	1.39E-08	0.334	1.39E-08	0.333
1.25E-06	0.359	1.25E-07	0.341	1.25E-08	0.334	1.25E-08	0.333

$r_w^2 S_1/(4T_1 t)$ decreases or as t increases. The rate of decrease, decreases with increasing time, denoting an asymptotic value of Q_1/Q_p for large time. In these cases, near asymptotic value of 0.341 was obtained at $r_w^2 S_1/(4T_1 t)$ = 1.25E-07. This qualitative nature is also observed for case-III and case-IV, where the asymptotic value of 0.333 was obtained at $r_w^2 S_1/(4T_1 t)$ = 4.17E-08. This asymptotic value show that at this time Q_1/Q_2 = 1/2 = T_1/T_2. Therefore, it is observed that after sufficiently long time Q_1/Q_2 approaches T_1/T_2. For cases I & II, asymptotic value of Q_1/Q_p is approached later than that observed in cases III & IV. Thus, higher the value of B_1/K_1, earlier observed is the asymptotic value of Q_1/Q_p.

It has been observed that the present numerical approach is capable of modelling flow to multi-aquifer well taping all aquifer-layers of multi-aquifer system. Above are shown the results for a two-aquifer system, which are in conformity with the general conclusions of Nautiyal(1984). The present approach can be extended for an aquifer system consisting of more than two aquifers. The main advantage of the present method over the one suggested by Nautiyal(1984) is "in the present, the spatial variations in the parameters of aquifer/aquitard can be taken into account while in Nautiyal method, these can not be taken into account".

CONCLUSION

A methodology has been developed to model the flow towards a fully penetrating well tapping two aquifers separated by aquitard using the three-dimensional ground water flow model (MODFLOW) and the discrete approach approach. This enables the use of MODFLOW for realistic modelling of flow to a multi-aquifer well. The main advantage of the approach is that the spatial variations in the parameters of aquifers/aquitard can be taken into account. The results obtained using the methodology are found in conformity to those available from analytical studies. Contribution of aquifer having higher diffusivity (T/S), increases with time while the contribution of aquifer having lower diffusivity, decreases with time. Q_1/Q_2 approaches T_1/T_2 as the steady state is approached. Higher the value of B_1/K_1, earlier observed is the asymptotic value of Q_1/Q_p which is equal to T_1/T_2. The methodology can easily be extended for an aquifer-system consisting of more than two aquifers.

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APPENDIX : USGS GROUND WATER FLOW MODEL (MODFLOW)

MODFLOW (MODular three dimensional finite difference FLOW model) was originally Developed by McDonald and Harbaugh, USGS, USA in 1984. It simulates three dimensional groundwater flow using a block-centred finite difference approach. It can simulate aquifer-layers as confined, unconfined or changing from unconfined to confined during the course of simulation. Stresses from external sources, such as, wells, areal recharge, evapotranspiration, leakage from drains and riverbeds, and flow from flow-controlled or head controlled boundaries. can be simulated individually or in combination through the modular structural of the model. It has two modules for the solution of finite difference flow equations, viz., Strongly Implicit Procedure(SIP) and Slice Successive Over-relaxation method(SSOR), either of which can be selected.

Governing Equation:

Boussinesq equation governing the three dimensional unsteady groundwater flow in a heterogeneous and anisotropic medium is,

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - W = S_s\frac{\partial h}{\partial t} \qquad \text{(A-11}$$

Where, K_{xx}, K_{yy}, and K_{zz} are the hydraulic conductivities along the x, y, and z axes respectively; h is the piezometric head; W is a volumetric flux per unit volume; S_s is the specific storage of the porous material; and t is the time.

Discretization:

For the formulation of finite difference equation, aquifer system is generally discretized into a number of elements called cells, the locations of which are described by indices denoting rows, column and layers. To conform with computer array convention, an i,j,k, indexing system is used. A discretized hypothetical system with 5 rows, 9 columns and 5 layers, is shown in fig. A-1. The width of the cells in row direction at a given column, j, is designated as Δr_j; width of cells in the column direction at a given row, i, is designated as Δc_i; and thickness of cells in a given layer, k, is designated Δv_k.

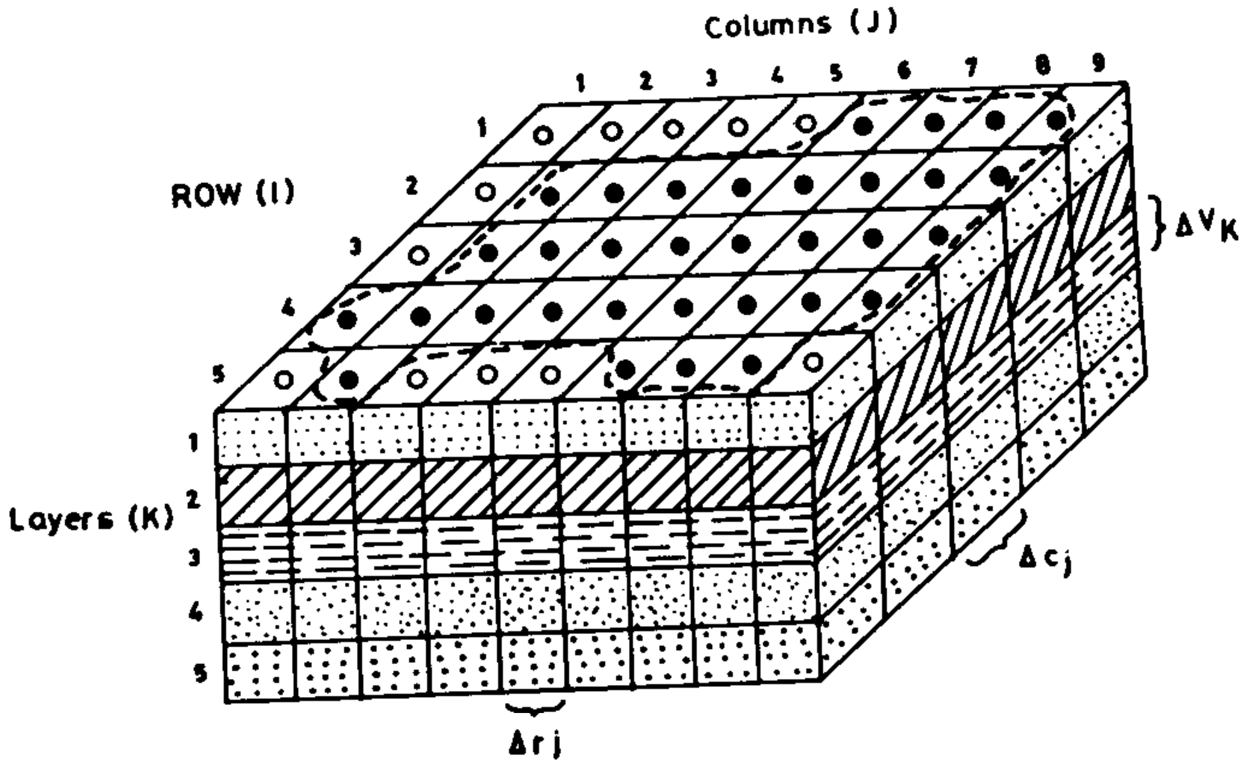
A simulation period is discretized into a number of stress periods and each stress period into a number of time steps. In each stress period, all the external stresses are assumed constant. To simulate the boundary conditions 'constant head cells' and 'inactive cells' are specified in advance. Constant head cells are those for which head is specified in advance, and is held at this particular value through all time step of simulation. Inactive cells are those no flow into or out of cell is permitted, in any time step of simulation. the remaining cells of the mesh, termed variable head cells are characterized by heads that are unspecified and free to vary with time.

Finite Difference Equations:

In the development of finite difference equation, water balance within a cell i,j,k using Darcy's law is obtained. Total internal groundwater flow to the cell i,j,k is the flow from all the six adjacent cells. The difference equation for the cell i,j,k in backward difference form as used in the model is,

$$CR_{i,j-1/2,k}[\, h^m_{i,j-1,k} - h^m_{i,j,k} \,] + CR_{i,j+1/2,k}[\, h^m_{i,j+1,k} - h^m_{i,j,k} \,] +$$

$$CC_{i-1/2,j,k}[\, h^m_{i-1,j,k} - h^m_{i,j,k} \,] + CC_{i+1/2,j,k}[\, h^m_{i+1,j,k} - h^m_{i,j,k} \,] +$$



LEGEND

– – – Aquifer Boundary
● Cell
○ Inactive cell

FIG. A·1: A DISCRETIZED HYPOTHETICAL SYSTEM

$$CV_{i,j,k-1/2}[h^m_{i,j,k-1} - h^m_{i,j,k}] + CV_{i,j,k+1/2}[h^m_{i,j,k+1} - h^m_{i,j,k}]$$

$$P_{i,j,k}h^m_{i,j,k} + Q_{i,j,k} = S_{s_{i,j,k}}(\Delta r_j \Delta c_i \Delta v_k)\frac{h^m_{i,j,k} - h^{m-1}_{i,j,k}}{t_m - t_{m-1}} \qquad \text{(A-2)}$$

Where,

$S_{s_{i,j,k}}$ = specific storage of cell i,j,k;

m = index denoting time step;

t_m = time at the end of m^{th} time-step;

t_{m-1} = time at the end of $(m-1)^{th}$ time-step;

$P_{i,j,k}$ = a constant.

$P_{i,j,k}h^m_{i,j,k} + Q_{i,j,k}$ represents total flow to the cell, i,j,k due to all external excitations. CR's, CC's, and CV's are the conductances in row, column., and vertical directions respectively. Conductance is defined as the product of hydraulic conductivity and the area through which the flow occurs divided by the length of flow path. An equation similar to eq. (A-2) can be written for each active cell of the system and thus, a set of equations can be formed for the system. This set of simultaneous equations is solved to get values of heads at all nodes.

<u>Simulation Packages:</u>

The modular structure of the model consist of a main program and a series of highly independent subroutines called 'modules'. The modules are grouped into 'packages'. Each package deals with a specific feature of the groundwater system. Four packages have been used for the present analysis brief description of which are given below.

The **'Basic Package'** handles a number of administrative tasks for the model. It reads data on the number of rows, columns, layers and stress periods, on the major options to be used, and on the location of input data for those options. it allocates space in the computer memory for model arrays; it reads data specifying initial and boundary conditions; it reads and implements data establishing the discretization of time, it sets up starting head arrays for each time step.

The **'Block-Centered Package'** computes the conductance components of the finite difference equation which determine flow between adjacent cells. It also computes the term which determines the rate and movement of water to and from the storage. To make the required calculations, it is assumed that a node is located at the centre of each model cell; thus the name Block-Centered Flow is given to the package. It calculates terms of finite difference equations which represents flow within porous medium, specifically, flow from cell to cell and flow into storage. The **'Strongly Implicit Procedure Package'** is a package for solving a large system of simultaneous linear equations by iteration.

The **'Well Package'** is designated to simulate features such as wells which withdraw water from the aquifer(or add water to it) at a specified rate during a given stress period, where the rates are independent of both the cell area and head in the cell. Well discharge is handled in the package by specifying the rate, Q, at which each individual well adds water to the aquifer or removes water from it, during each stress period of simulation. Negative values of Q are used to indicate well discharge, while positive values of Q indicate a recharging well. The well Package, as it is presently formulated in the model, does not accommodate wells, which are open to more than one layers of the aquifer. However, a well of this type can be represented as a group of single layer wells, each open to only one of the layers tapped by the multi-layered well, and each having an individual Q term specified for each stress period. This approach to represent a multi-layer well fails to take into account interconnection between various layers provided by the well itself and is thus an incomplete representation of the problem.

The **'Evapotranspiration Package'** simulates the effects of plant transpiration and direct evaporation in removing water from the saturated groundwater regime. The purpose of **'River Package'** is to simulate the effects of flow between surface-water features and ground-water system. The **'Recharge Package'** is designed to simulate areally distributed recharge to groundwater system. Most commonly, areal recharge occurs as a result of precipitation that percolates the groundwater system. The **'Drain Package'** is designed to simulate the effects of features such as agricultural drains, which remove water from the aquifer at a rate proportional to the difference between the head in the aquifer and some fixed head or elevation, so long as the head in the aquifer is above that elevation, but which have no effect if head falls below that level.

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